Pure Math 30:

TRIGONOMETRY 1

$$90° = \frac{\pi}{2}$$
$\{0,1\}$

$120° = \frac{2\pi}{3}$ $\left\{-\frac{1}{2}, \frac{\sqrt{3}}{2}\right\}$ $60° = \frac{\pi}{3}$ $\left\{\frac{1}{2}, \frac{\sqrt{3}}{2}\right\}$

$135° = \frac{3\pi}{4}$ $\left\{-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right\}$ $45° = \frac{\pi}{4}$ $\left\{\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right\}$

$150° = \frac{5\pi}{6}$ $\left\{-\frac{\sqrt{3}}{2}, \frac{1}{2}\right\}$ $30° = \frac{\pi}{6}$ $\left\{\frac{\sqrt{3}}{2}, \frac{1}{2}\right\}$

$180 = \pi$ $\{-1,0\}$ $0° = 360° = 2\pi$ $\{1,0\}$

$210° = \frac{7\pi}{6}$ $\left\{-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right\}$ $330° = \frac{11\pi}{6}$ $\left\{\frac{\sqrt{3}}{2}, -\frac{1}{2}\right\}$

$225° = \frac{5\pi}{4}$ $\left\{-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right\}$ $315° = \frac{7\pi}{4}$ $\left\{\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right\}$

$240° = \frac{4\pi}{3}$ $\left\{-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right\}$ $300° = \frac{5\pi}{3}$ $\left\{\frac{1}{2}, -\frac{\sqrt{3}}{2}\right\}$
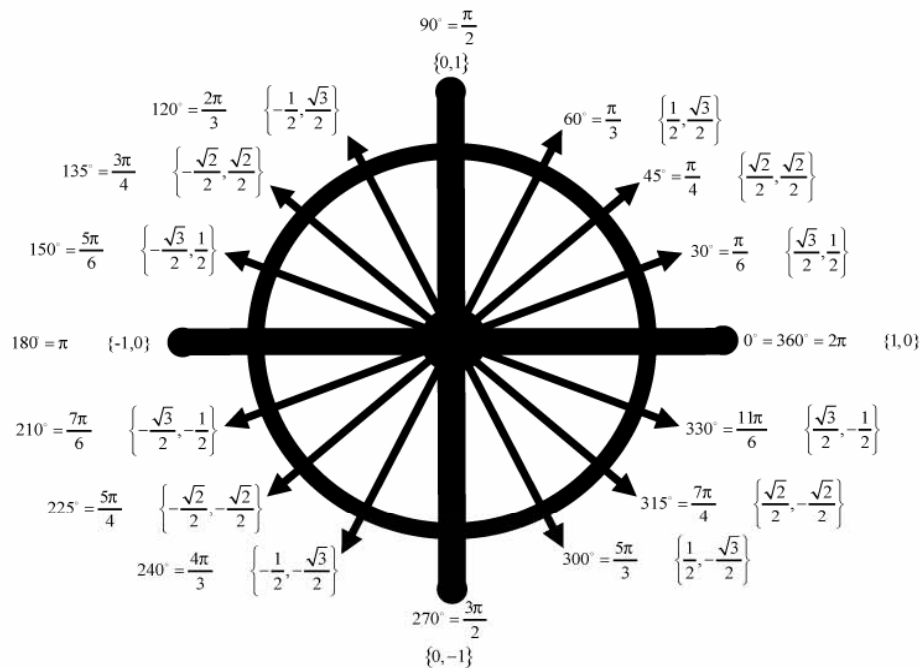
$$270° = \frac{3\pi}{2}$$
$\{0,-1\}$

LESSON FOUR

Graphing a & d
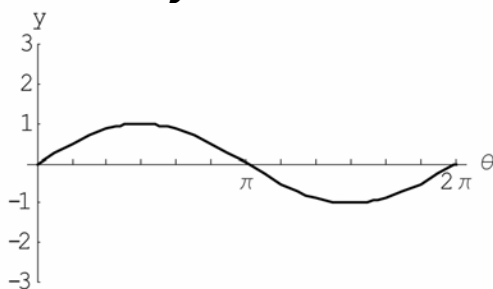
Pure Math 30: **EXPLAINED!**

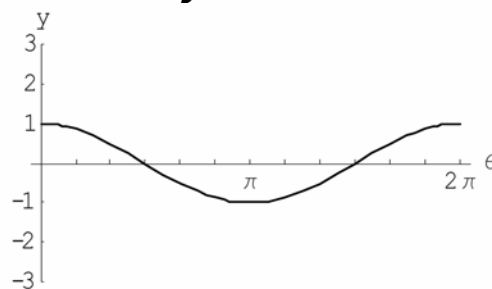By Barry Mabillard

TRIGONOMETRY LESSON 4
PART I AMPLITUDE

The first step in learning how to graph trigonometric functions is to look at the basic sine & cosine graphs. Make special note of the differences and similarities between them.

Memorize these graphs, as they serve the basis for the rest of this unit.

$$y = \sin\theta \qquad\qquad\qquad\qquad y = \cos\theta$$

Similarities:

- Look at the y-axis. Both graphs have a maximum at 1, and a minimum at -1.
- Look at the x-axis. Both graphs start at 0 and end at 2π.
  (One complete revolution of the unit circle)
- The midline for each graph is at y = 0.

Differences:

- Notice how the sine pattern looks like a wave, while the cosine pattern looks like a U.

How to graph in degree mode:

Click MODE on your calculator, then select "degree"

Set up your window using the window settings on the right.
You will adjust them later for more complicated graphs.

Degree mode is a practical way of graphing. All values
will be given in degrees, so it is easy _(now that you have memorized the unit circle)_ to immediately convert from degrees to radian fractions.
Degree graphs are primarily used for basic drawing of trig functions.

> X min = 0
> X max = 360
> X scl = 90
> Y min = -1
> Y max = 1
> Y scl = 1

How to graph in radian mode:

Click MODE on your calculator, then select "radian"

Use the window settings on the right

When you try to get results from your calculator, all angle answers will be given as decimal radians. This can be inconvenient, since most answers expected of you will be either degrees or radian fractions.
Radian graphs are primarily used for application questions.

> X min = 0
> X max = 2π
> X scl = $\pi/2$
> Y min = -1
> Y max = 1
> Y scl = 1

PURE MATH 30: EXPLAINED!
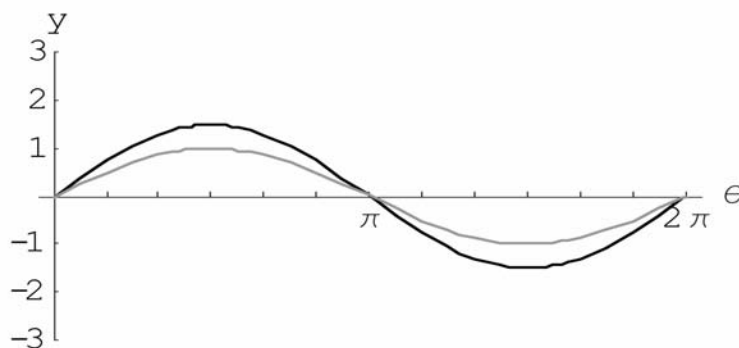
TRIGONOMETRY LESSON 4
PART I AMPLITUDE

The distance from the midline of a trig graph to the maximum or minimum is called the amplitude.

Amplitude of 6 units

Amplitude of 6 units
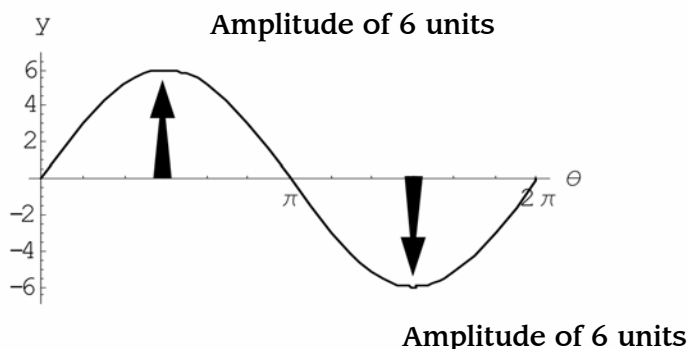
The amplitude is the number you multiply all y-values by to do a vertical stretch.

Black: y = 1.5sinθ
Grey: y = sinθ

The amplitude may be found from a graph by using the formula:

$$\text{Amplitude} = \frac{|\text{max - min}|}{2}$$

This formula is useful in application questions where a graph is hard to read.

For example, the calculation in the previous graph is:

$$\text{Amplitude} = \frac{|1.5 - (-1.5)|}{2} = \frac{3}{2} = 1.5$$

Memorize the amplitude formula, as it is not on your formula sheet!

PURE MATH 30: EXPLAINED!

Coefficients with a magnitude greater than one will stretch the graph, making it taller.
Coefficients with a magnitude less than one will compress the graph, making it shorter.

$$y = 3\sin\theta$$

$$y = \frac{1}{2}\cos\theta$$
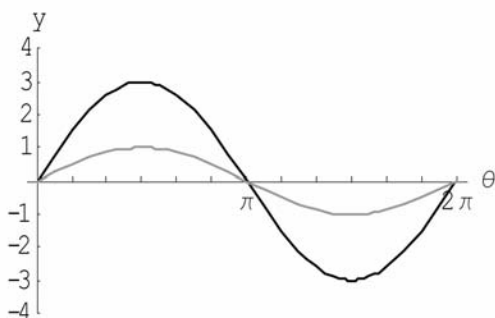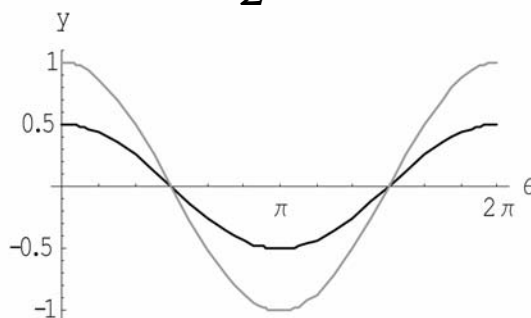
Grey: $y = \sin\theta$
Black: $y = 3\sin\theta$

Grey: $y = \cos\theta$
Black: $y = \frac{1}{2}\cos\theta$

If there is a negative sign in front of a trig function, this will flip the graph upside down in addition to whatever stretch is required.

$$y = -3\sin\theta$$

$$y = -\frac{1}{2}\cos\theta$$
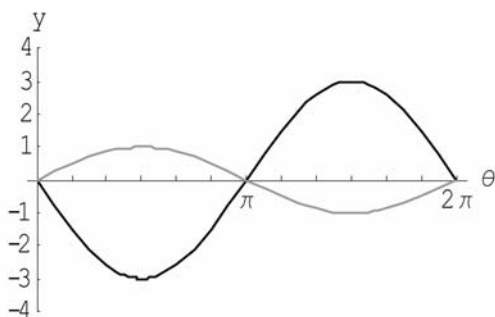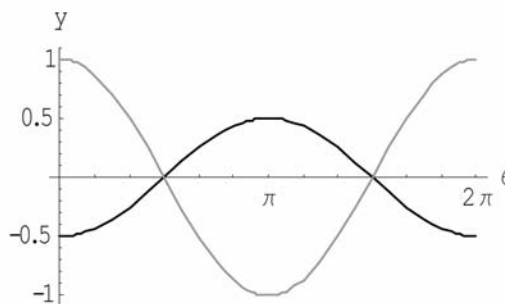
Grey: $y = \sin\theta$
Black: $y = -3\sin\theta$

Grey: $y = \cos\theta$
Black: $y = -\frac{1}{2}\cos\theta$

Terminology Alert
The term amplitude is defined as the absolute value of the coefficient.
For $y = -3\sin\theta$, we would say *"the graph has an amplitude of 3, and is reflected in the x-axis."* It would be incorrect to say *"the graph has an amplitude of -3."*

PURE MATH 30: EXPLAINED!

TRIGONOMETRY LESSON 4
PART I AMPLITUDE

QUESTIONS: *Draw the following graphs by hand, then graph them in your calculator to check.*

1) $y = -\dfrac{1}{2}\cos\theta$
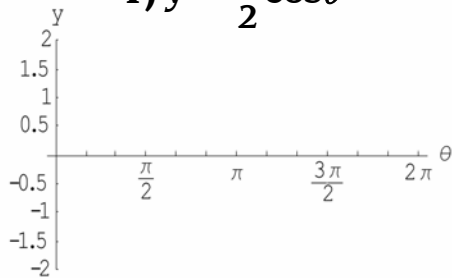
2) $y = 2\cos\theta$
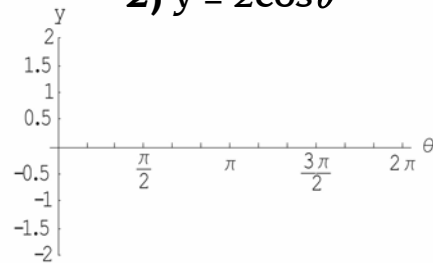
3) $y = -3\sin\theta$
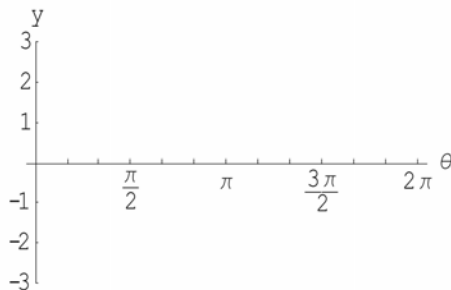
4) $y = -\dfrac{1}{2}\sin\theta$
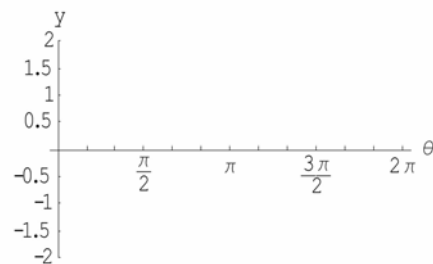
For each of the following graphs, write the equation:

5)

6)

7)

8)
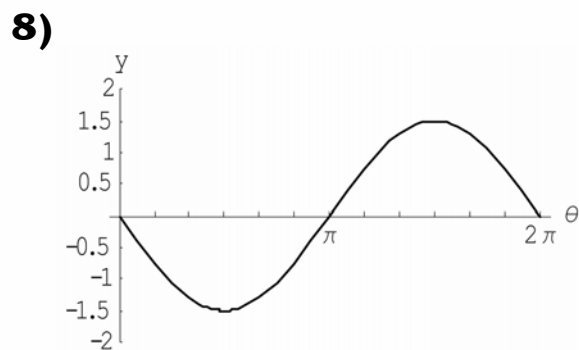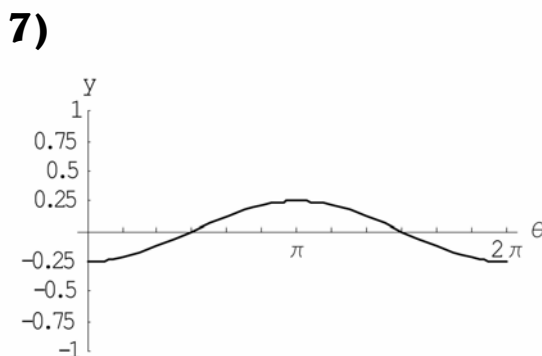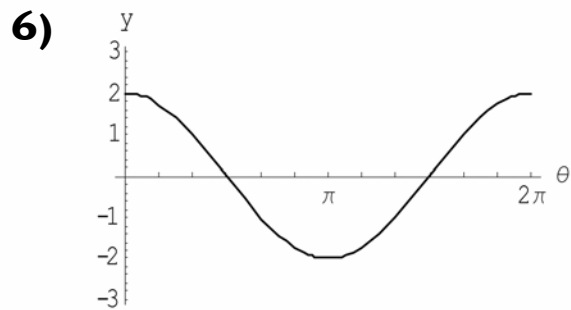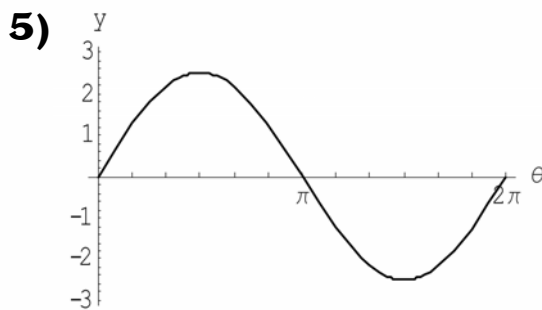
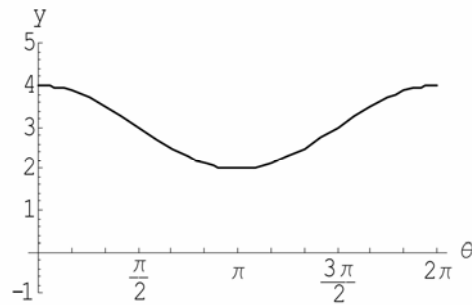# TRIGONOMETRY LESSON 4
PART I   AMPLITUDE

Answers:

1)
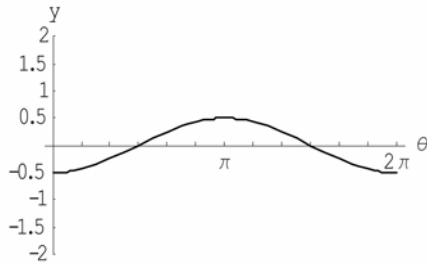
2)
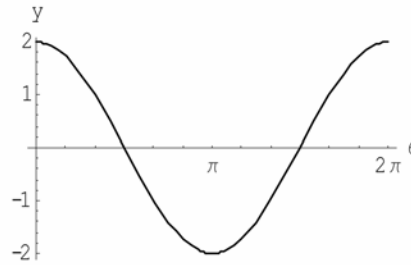
3)
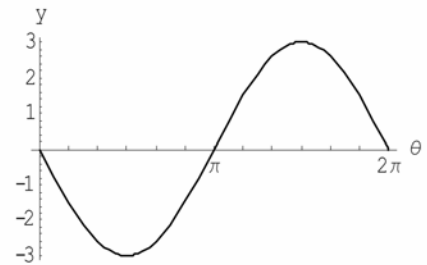
4)
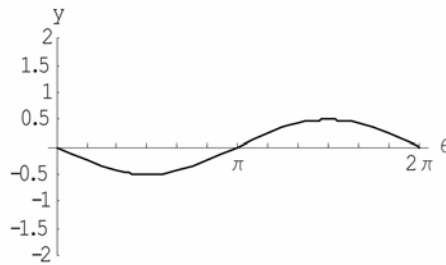
5) $y = 2.5\sin\theta$

6) $y = 2\cos\theta$

7) $y = -0.25\cos\theta$

8) $y = -1.5\sin\theta$

TRIGONOMETRY LESSON 4
PART II VERTICAL TRANSLATION

The vertical translation of a trig graph is how far up or down we shift the midline.

Example 1: Graph $y = \sin\theta + 3$
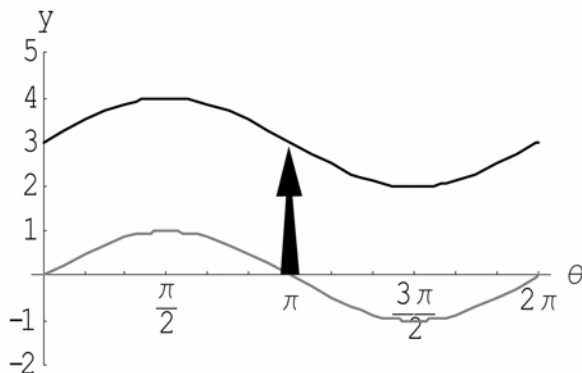
Grey: $y = \sin\theta$
Black: y = $\sin\theta$ +3

Example 2: Graph $y = \cos\theta - 5$
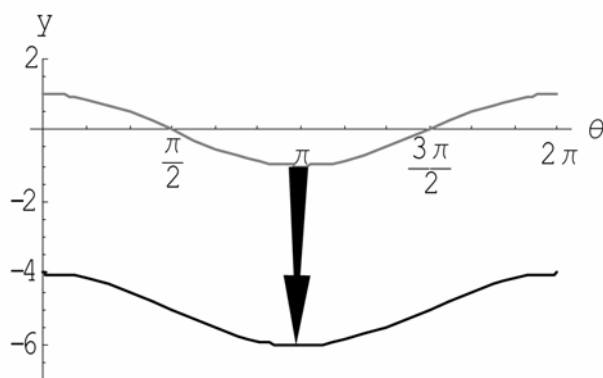
Grey: $y = \cos\theta$
Black: y = $\cos\theta$ - 5

There is an equation you can use for finding the equation of the midline:

$$y = \frac{\min + \max}{2}$$

From the graph in Example 2: ($y = \cos\theta - 5$),
The minimum is -6 and the maximum is -4.

$$y = \frac{-6 + (-4)}{2} = \frac{-10}{2} = -5$$

Using the formula on the graph verifies that
the midline equation is y = -5
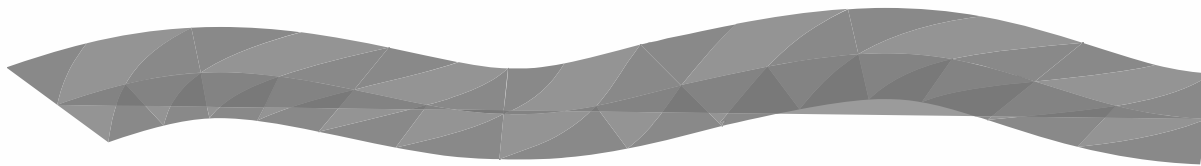
TRIGONOMETRY LESSON 4
PART II   VERTICAL TRANSLATION

Questions: *Draw the graph:*

1) $y = \sin\theta - 1$
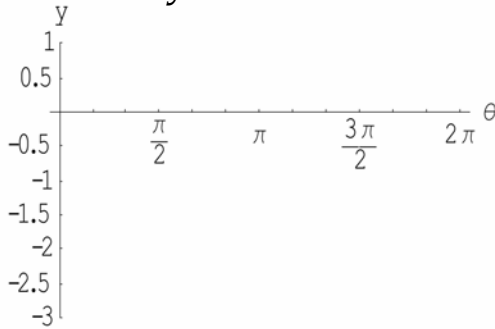
2) $y = \cos\theta + 2$
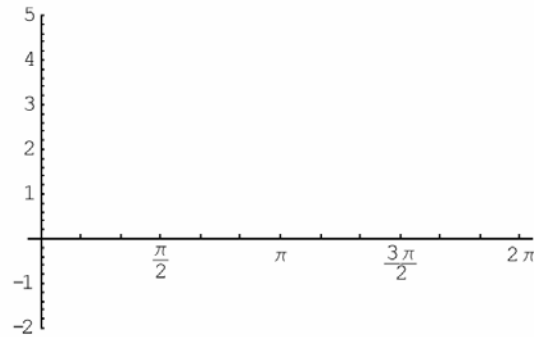
3) $y = \cos\theta - 3$
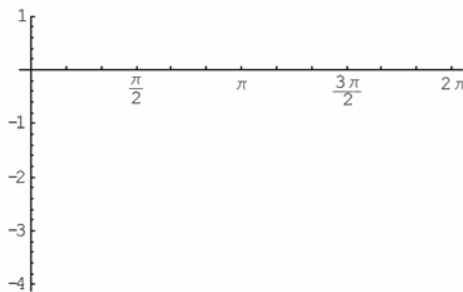
4) $y = \sin\theta + 4$
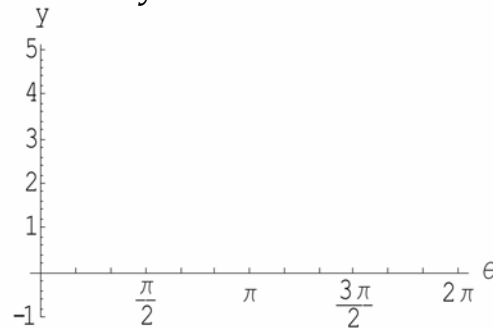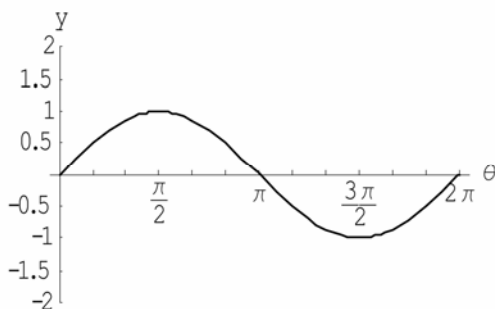
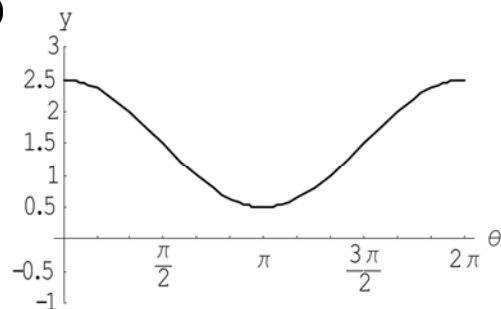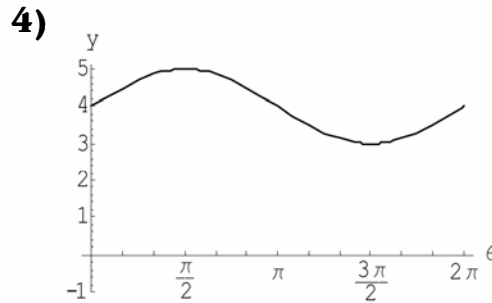*For each of the following graphs, write the equation.*

5)

6)

7)

8)
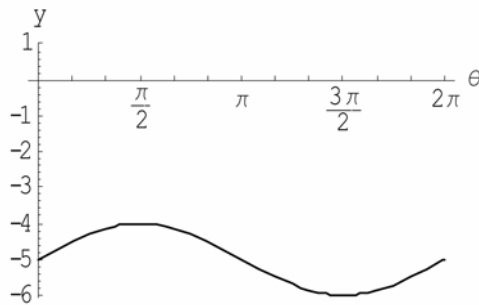
TRIGONOMETRY LESSON 4
PART II VERTICAL TRANSLATION

Answers:

1)

2)

3)

4)
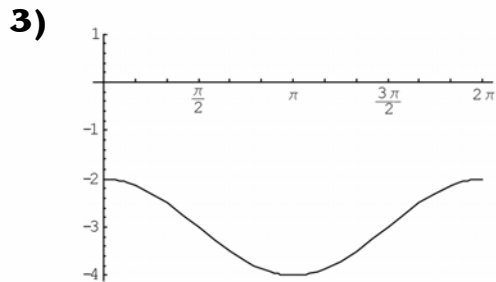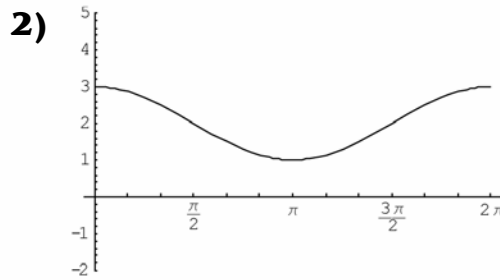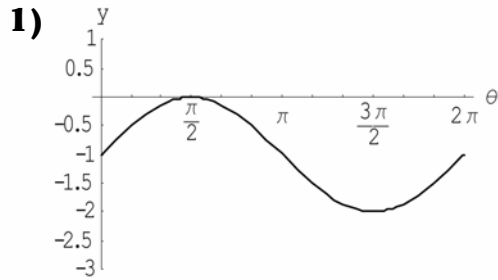
5) $y = \sin\theta - 5$

6) $y = \cos\theta + 3$

7) $y = \sin\theta$

8) $y = \cos\theta + \dfrac{3}{2}$

TRIGONOMETRY LESSON 4
PART III   GRAPHING A AND D

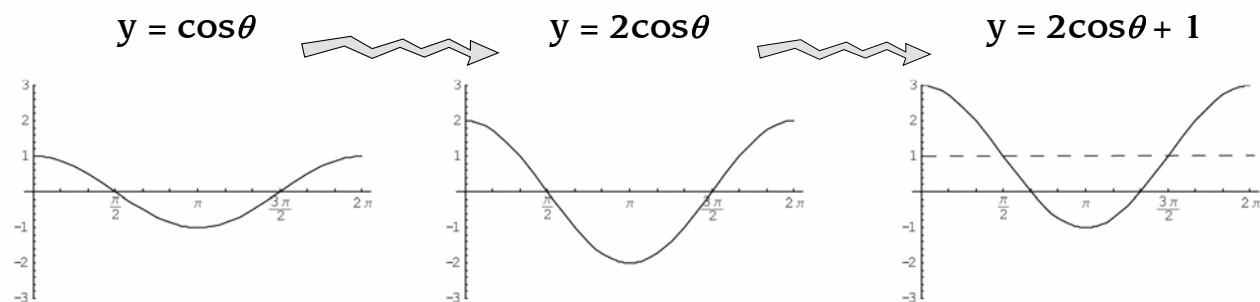**We will now look at trig graphs with the form:** $y = a\sin\theta + d$

"**a**" is the letter used to represent amplitude.
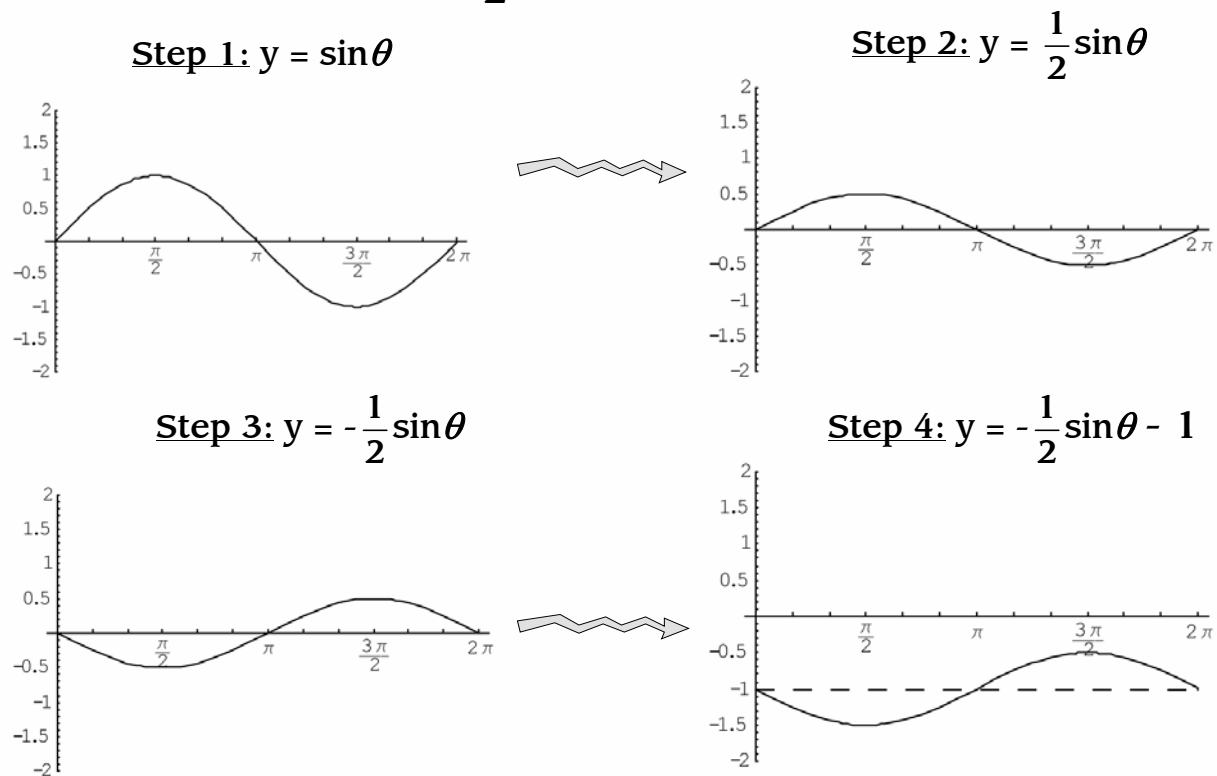"**d**" is the letter used to represent vertical translation.

When doing the transformation, we should follow a particular order.
First apply the amplitude, then the vertical translation.

__Example 1:__ Graph y = 2cosθ + 1

y = cosθ y = 2cosθ y = 2cosθ + 1

__Example 2:__ Graph y = $-\dfrac{1}{2}$sinθ - 1

__Step 1:__ y = sinθ __Step 2:__ y = $\dfrac{1}{2}$sinθ

__Step 3:__ y = $-\dfrac{1}{2}$sinθ __Step 4:__ y = $-\dfrac{1}{2}$sinθ - 1
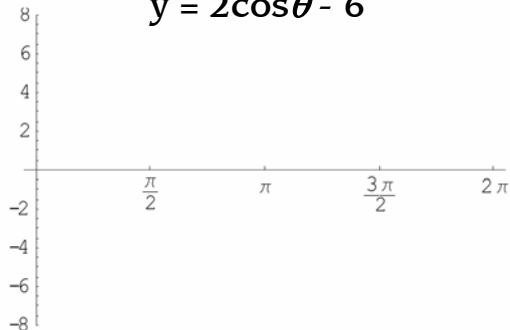
TRIGONOMETRY LESSON 4
PART III   GRAPHING A AND D
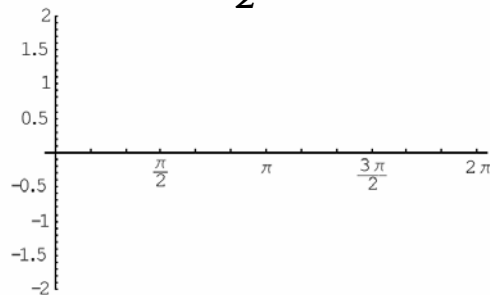
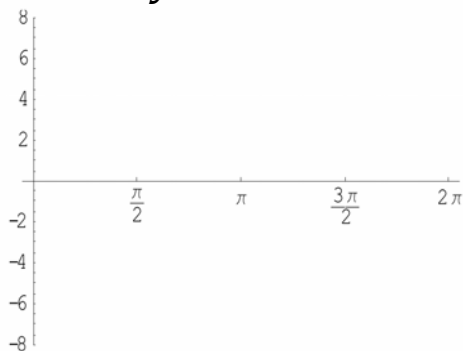**Questions:** Draw the graph.

1) $y = 2\cos\theta - 6$

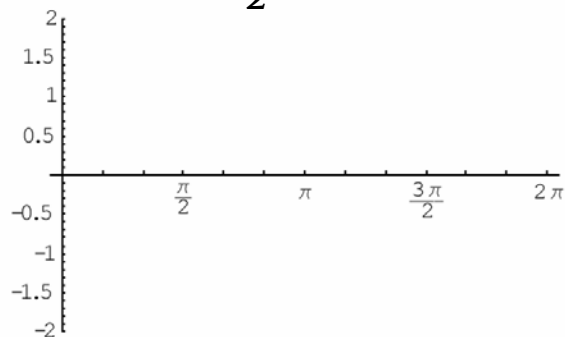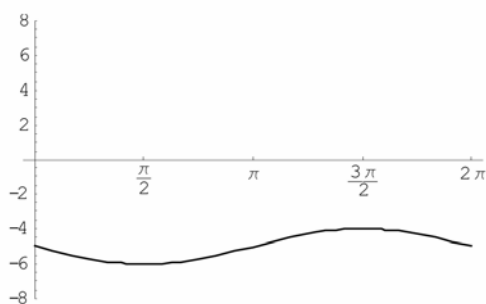**2)** $y = -\dfrac{1}{2}\cos\theta - 1$

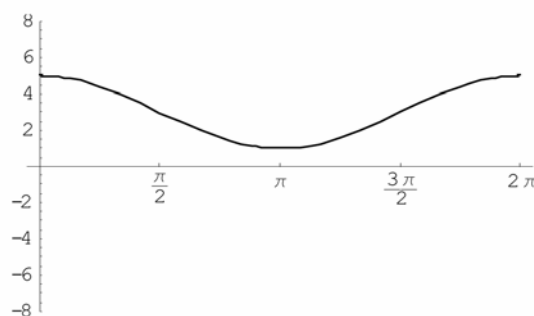**3)** $y = 3\sin\theta + 1$

4) $y = -\dfrac{1}{2}\sin\theta + 1$

For each of the following graphs, write the equation.

5)

6)

7)
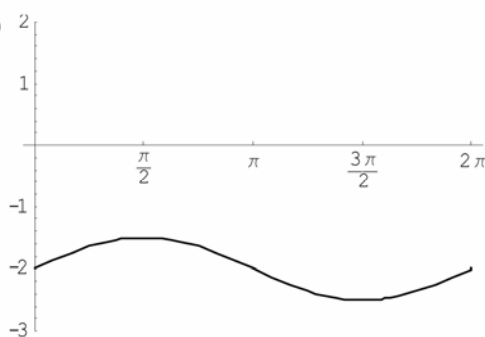
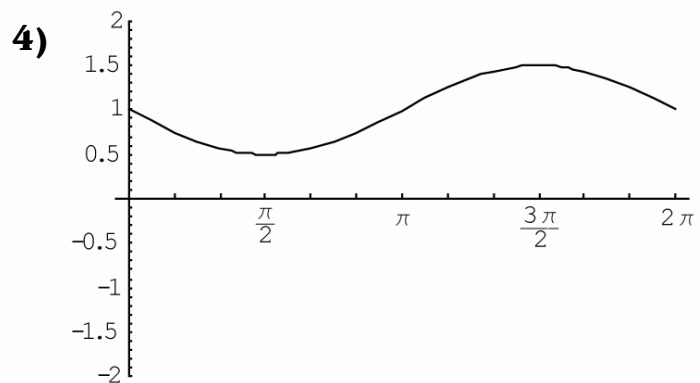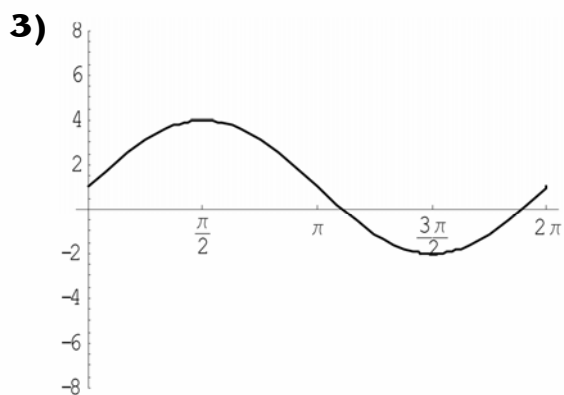# TRIGONOMETRY LESSON 4
PART III   GRAPHING A AND D

Answers:

1)

2)

3)

4)

5) $y = -\sin\theta - 5$

6) $y = 2\cos\theta + 3$

7) $y = \dfrac{1}{2}\sin\theta - 2$