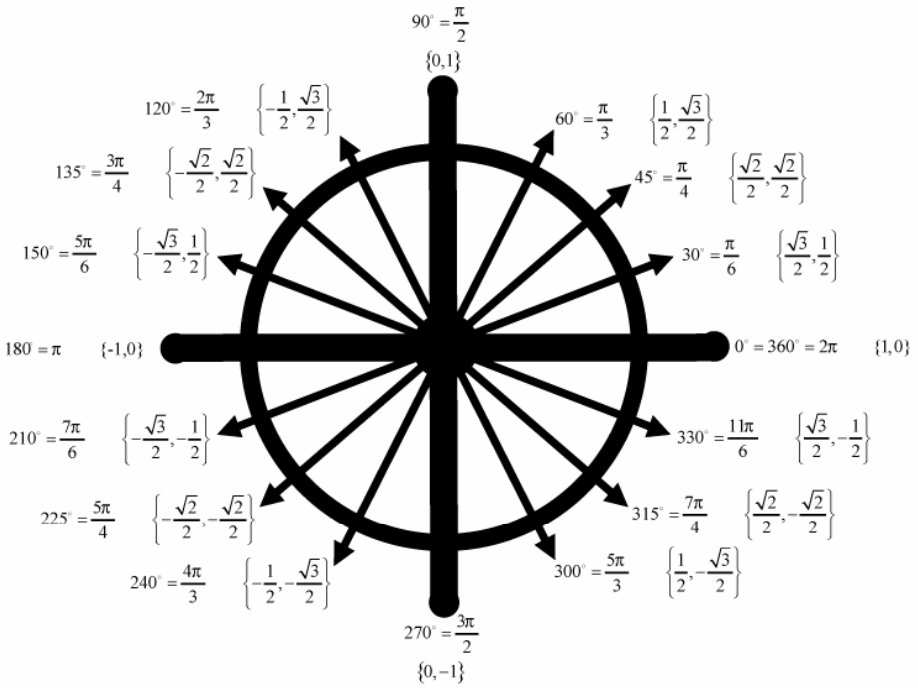
Pure Math 30:

TRIGONOMETRY I



LESSON THREE

Six Trigonometric Ratios

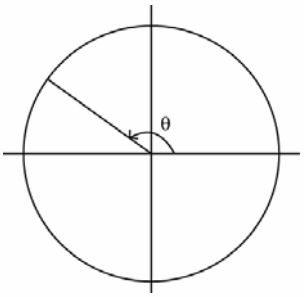
Pure Math 30: EXPLAINED!

By Barry Mabillard

TRIGONOMETRY LESSON 3

PART 1 FINDING QUADRANTS

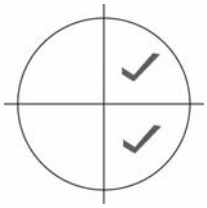
Look at the angle shown in the diagram to the right.
Is it possible to predict if $\sin\theta$ or $\cos\theta$ will be positive or negative?



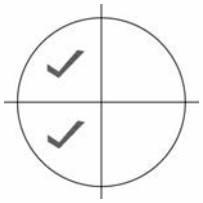
The answer is yes, and you can predict the sign by knowing which quadrant the terminal arm is located in.

Memorize the following rules:

$\cos\theta$ is positive in quadrants I & IV
(where x is positive.)



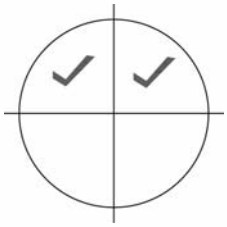
$\cos\theta$ is negative in quadrants II & III
(where x is negative.)



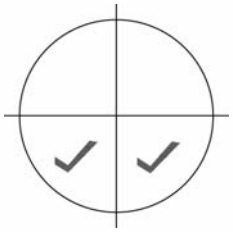
You can easily verify these rules in your calculator by taking the cosine of different angles.

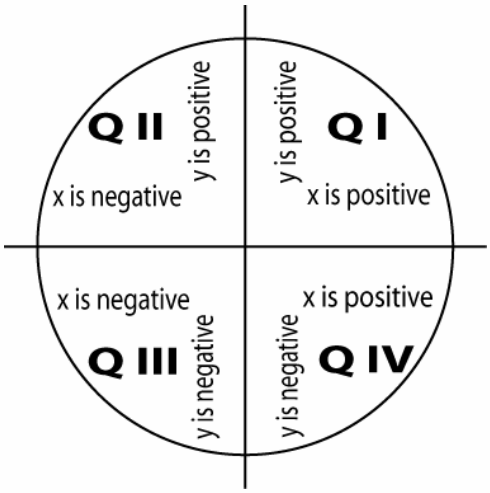
QI : $\cos 60^0 = 0.5$
QII : $\cos 120^0 = -0.5$
QIII : $\cos 240^0 = -0.5$
QIV : $\cos 300^0 = 0.5$

$\sin\theta$ is positive in quadrants I & II
(where y is positive.)

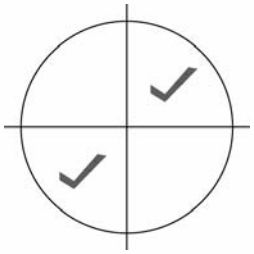


$\sin\theta$ is negative in quadrants III & IV
(*where y is negative.)*

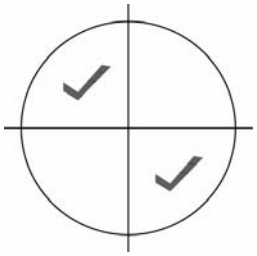




$\tan\theta$ is positive in quadrants I & III
(x & y have the same signs.)



$\tan\theta$ is negative in quadrants II & IV
(x & y have different signs.)



The signs of the reciprocal trig ratios can be determined by remembering the following:

$\sec\theta$ follows the same pattern as $\cos\theta$

$\csc\theta$ follows the same pattern as $\sin\theta$

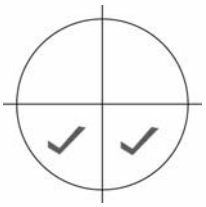
$\cot\theta$ follows the same pattern as $\tan\theta$

TRIGONOMETRY LESSON 3

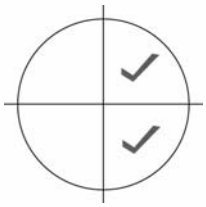
PART 1 FINDING QUADRANTS

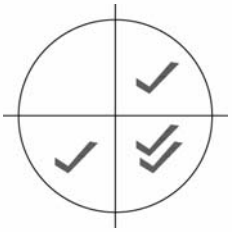
Example 1: $\sin\theta < 0$ & $\cos\theta > 0$. What quadrant is the angle in?

$\sin\theta$ is negative in these quadrants



$\cos\theta$ is positive in these quadrants

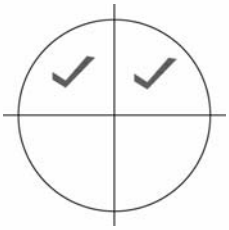




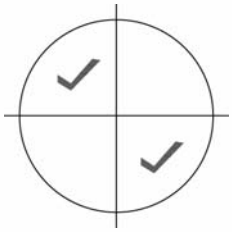
We have overlap in quadrant IV, so that is where the angle is.

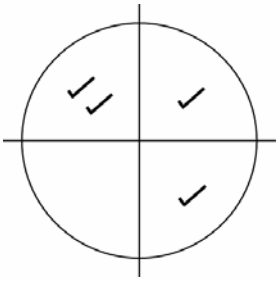
Example 2: $\csc\theta > 0$ & $\cot\theta = -\frac{\sqrt{3}}{3}$. What quadrant is the angle in?

$\csc\theta$ follows the same pattern as $\sin\theta$, and $\sin\theta$ is positive in these quadrants



$\cot\theta$ follows the same pattern as $\tan\theta$, and $\tan\theta$ is negative in these quadrants





We have overlap in quadrant II, so that is where the angle is.

Even though an actual value is given for $\cot\theta$, it is not needed if we only want to find the quadrant.

We do need the negative though, since that indicates $\cot\theta < 0$.

TRIGONOMETRY LESSON 3

PART I FINDING QUADRANTS

***Questions:** For each of the following, state what quadrant the angle is in:*

1) $\sin\theta > 0$ & $\cos\theta < 0$

2) $\csc\theta > 0$ & $\tan\theta > 0$

3) $\cot\theta < 0$ & $\sec\theta > 0$

4) $\sec\theta = -2$ & $\tan\theta < 0$

5) $\csc\theta = \frac{1}{5}$ & $\cos\theta = -\frac{5}{6}$

6) $\cos\theta > 0$ & $\tan\theta < 0$

7) $\sin\theta < 0$ & $\sec\theta > 0$

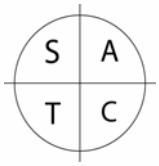
8) $\sin\theta > 0$ & $\tan\theta > 0$

9) $\cos\theta = -0.2$ & $\cot\theta < 0$

10) $\cot\theta > 0$ & $\sin\theta < 0$

11) $\cot\theta < 0$ & $\csc\theta = \frac{\sqrt{3}}{3}$

12) $\csc\theta < 0$ & $\sec\theta < 0$



An alternative way of doing these questions is to use the **CAST** rule.

C: $\cos\theta$ (and $\sec\theta$) is positive, everything else is negative.
A: all are positive
S: $\sin\theta$ (and $\csc\theta$) is positive, everything else is negative.
T: $\tan\theta$ (and $\cot\theta$) is positive, everything else is negative.

If you use this method, remember that you start in quadrant IV and spell it out counterclockwise.

Answers:

1) II
2) I
3) IV
4) II
5) II
6) IV
7) IV
8) I
9) II
10) III
11) II
12) III

TRIGONOMETRY LESSON 3

PART II SIX TRIGONOMETRIC RATIOS

The six trigonometric ratios are shown to the right. Given a point, it is possible to evaluate all six ratios by creating a triangle and determining the lengths of the three sides.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

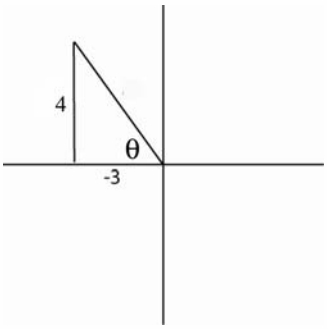
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

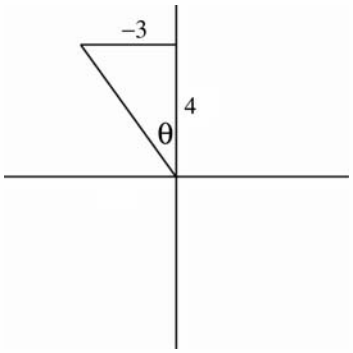
Example 1: Given the point P(-3, 4), find the six trigonometric ratios.

1) Draw your terminal arm from the origin to the given point. Form a triangle by drawing a line from the point to the closest ***x-axis***!

Draw like this.

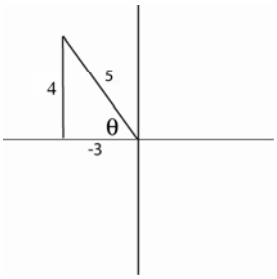


***NOT* like this**



2) Use Pythagoras to find the unknown side:

$$a^2 + b^2 = c^2$$
$$(-3)^2 + 4^2 = c^2$$
$$9 + 16 = c^2$$
$$25 = c^2$$
$$c = 5$$



*The hypotenuse is ALWAYS positive!

3) Now that all three sides are known, list the trigonometric ratios.

$$\sin\theta = \frac{4}{5} \qquad \csc\theta = \frac{5}{4}$$

$$\cos\theta = \frac{-3}{5} \qquad \sec\theta = \frac{5}{-3}$$

$$\tan\theta = \frac{4}{-3} \qquad \cot\theta = \frac{-3}{4}$$

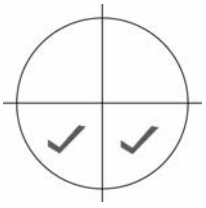
TRIGONOMETRY LESSON 3

PART II SIX TRIGONOMETRIC RATIOS

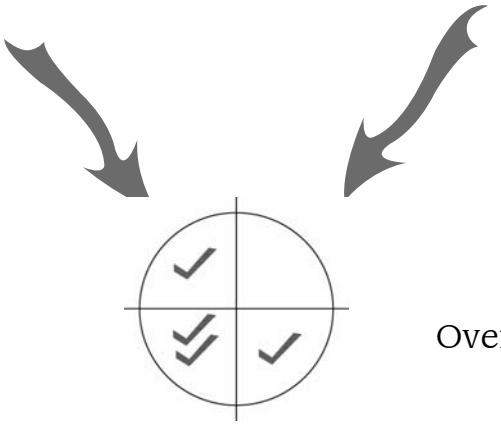
Example 2: If $\csc\theta = -\frac{13}{5}$ and $\cos\theta < 0$, find the six trigonometric ratios.

1) First find the quadrant the angle is in:

$\csc\theta$ follows the same pattern as $\sin\theta$, and $\sin\theta$ is negative in these quadrants

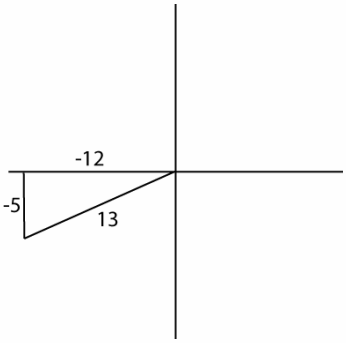


$\cos\theta$ is negative in these quadrants



Overlap in quadrant III.

2) Draw the triangle, and find the unknown side using Pythagoras:



$$a^2 + b^2 = c^2$$
$$(a)^2 + (-5)^2 = 13^2$$
$$a^2 + 25 = 169$$
$$a^2 = 144$$
$$a = 12$$

3) Now find the six trigonometric ratios.

$$\sin\theta = \frac{-5}{13} \qquad \csc\theta = \frac{13}{-5}$$

$$\cos\theta = \frac{-12}{13} \qquad \sec\theta = \frac{13}{-12}$$

$$\tan\theta = \frac{-5}{-12} = \frac{5}{12} \qquad \cot\theta = \frac{12}{5}$$

Even though the answer of 12 from Pythagoras is positive, you must include a minus sign since the number is being placed on the negative *x*-axis. Remember to label all triangle sides with the correct sign!

TRIGONOMETRY LESSON 3

PART II SIX TRIGONOMETRIC RATIOS

Questions: *For each of the following questions, state the six trigonometric ratios:*

1) $P(\sqrt{3}, -1)$

*Don't forget to rationalize the denominator when there is a radical on the bottom!

2) $P(-2, \sqrt{5})$

3) $P(-2, -3)$

4) $\cos\theta = -\frac{1}{3}$ & $\sin\theta > 0$

5) $\sec\theta = -\frac{5}{2}$ & $\tan\theta < 0$

6) $\csc\theta = -\frac{5}{4}$ & $\cot\theta > 0$

7) $\cot\theta = \frac{2}{5}$ & $\csc\theta < 0$

8) $\sin\theta = \frac{1}{2}$ & $\sec\theta > 0$

9) $\tan\theta = \sqrt{3}$ & $\cos\theta < 0$

TRIGONOMETRY LESSON 3

PART II SIX TRIGONOMETRIC RATIOS

$\sin\theta = \frac{-1}{2}$ $\quad \csc\theta = -2$

1) $\cos\theta = \frac{\sqrt{3}}{2}$ $\quad \sec\theta = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\tan\theta = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$ $\quad \cot\theta = -\sqrt{3}$

$\sin\theta = \frac{\sqrt{5}}{3}$ $\quad \csc\theta = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$

2) $\cos\theta = \frac{-2}{3}$ $\quad \sec\theta = \frac{3}{-2}$

$\tan\theta = \frac{\sqrt{5}}{-2}$ $\quad \cot\theta = \frac{-2}{\sqrt{5}} = \frac{-2\sqrt{5}}{5}$

$\sin\theta = \frac{-3}{\sqrt{13}} = \frac{-3\sqrt{13}}{13}$ $\quad \csc\theta = \frac{\sqrt{13}}{-3}$

3) $\cos\theta = \frac{-2}{\sqrt{13}} = \frac{-2\sqrt{13}}{13}$ $\quad \sec\theta = \frac{\sqrt{13}}{-2}$

$\tan\theta = \frac{-3}{-2} = \frac{3}{2}$ $\quad \cot\theta = \frac{2}{3}$

4) The angle is in quadrant II

$\sin\theta = \frac{2\sqrt{2}}{3}$ $\quad \csc\theta = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$

$\cos\theta = \frac{-1}{3}$ $\quad \sec\theta = -3$

$\tan\theta = \frac{2\sqrt{2}}{-1} = -2\sqrt{2}$ $\quad \cot\theta = \frac{-1}{2\sqrt{2}} = \frac{-\sqrt{2}}{4}$

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5) The angle is in quadrant II

$\sin\theta = \frac{\sqrt{21}}{5}$ $\quad \csc\theta = \frac{5}{\sqrt{21}} = \frac{5\sqrt{21}}{21}$

$\cos\theta = \frac{-2}{5}$ $\quad \sec\theta = \frac{5}{-2}$

$\tan\theta = \frac{\sqrt{21}}{-2}$ $\quad \cot\theta = \frac{-2}{\sqrt{21}} = \frac{-2\sqrt{21}}{21}$

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6) The angle is in quadrant II

$\sin\theta = \frac{-4}{5}$ $\quad \csc\theta = \frac{5}{-4}$

$\cos\theta = \frac{-3}{5}$ $\quad \sec\theta = \frac{5}{-3}$

$\tan\theta = \frac{-4}{-3} = \frac{4}{3}$ $\quad \cot\theta = \frac{3}{4}$

7) The angle is in quadrant III

$\sin\theta = \frac{-5}{\sqrt{29}} = \frac{-5\sqrt{29}}{29}$ $\quad \csc\theta = \frac{\sqrt{29}}{-5}$

$\cos\theta = \frac{-2}{\sqrt{29}}$ $\quad \sec\theta = \frac{-2\sqrt{29}}{29}$

$\tan\theta = \frac{-5}{-2} = \frac{5}{2}$ $\quad \cot\theta = \frac{2}{5}$

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8) The angle is in quadrant I

$\sin\theta = \frac{1}{2}$ $\quad \csc\theta = 2$

$\cos\theta = \frac{\sqrt{3}}{2}$ $\quad \sec\theta = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\tan\theta = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ $\quad \cot\theta = \frac{\sqrt{3}}{1} = \sqrt{3}$

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9) The angle is in quadrant III

$\sin\theta = \frac{-\sqrt{3}}{2}$ $\quad \csc\theta = \frac{2}{-\sqrt{3}} = \frac{2\sqrt{3}}{-3}$

$\cos\theta = \frac{-1}{2}$ $\quad \sec\theta = -2$

$\tan\theta = \frac{-\sqrt{3}}{-1} = \sqrt{3}$ $\quad \cot\theta = \frac{-1}{-\sqrt{3}} = \frac{\sqrt{3}}{3}$