แบบฝึกหัดชุดที่ 3
อนุพันธ์ของฟังก์ชันอดิศัย

1. จงหาอนุพันธ์ของฟังก์ชันต่อไปนี้ (ฟังก์ชันเอกซ์โพเนนเชียลและฟังก์ชันลอการิทึม)

(1) $y=\ln(7x^3-x+2)$ $\left[\frac{21x^2-1}{7x^3-x+2}\right]$

(2) $y=\ln\sqrt[3]{\frac{x-1}{x}}$ $\left[\frac{1}{3x(x-1)}\right]$

(3) $y=\ln(\sqrt{x+1}-\sqrt{x})$ $\left[-\frac{1}{2\sqrt{x(x+1)}}\right]$

(4) $y=\ln(x^2\sqrt{4-x^2})$ $\left[\frac{16-7x^3}{2x(4-x^3)}\right]$

(5) $y=\ln(\sqrt{x^2-1}-\sqrt{2x-6})$ $\left[\left(\frac{1}{\sqrt{x^2-1}-\sqrt{2x-6}}\right)\left(\frac{x}{\sqrt{x^2-1}}-\frac{1}{\sqrt{2x-6}}\right)\right]$

(6) $y=\log(3x)$ $\left[\frac{1}{x\ln 10}\right]$

(7) $y=\log_5(x^3+2x)$ $\left[\frac{3x^2+2}{(x^3+2x)\ln 5}\right]$

(8) $y=\ln(4x-5)$ $\left[\frac{4}{4x-5}\right]$

(9) $y=\ln\sqrt{3-x^2}$ $\left[\frac{x}{x^2-3}\right]$

(10) $y=(\ln x)^2$ $\left[\frac{2\ln x}{x}\right]$

(11) $y=\ln(x^2+x-1)^3$ $\left[\frac{6x+3}{x^2+x-1}\right]$

(12) $y=x\ln x-x$ $[\ln x]$

(13) $y=\ln\left(\frac{x}{1+x^2}\right)$ $\left[\frac{1-x^2}{x(1+x^2)}\right]$

(14) $y=\frac{\ln x^2}{x^2}$ $\left[\frac{2-4\ln x}{x^3}\right]$

(15) $y=\sqrt{\ln x}$ $\left[\frac{1}{2x\sqrt{\ln x}}\right]$

(16) $y=x^3\ln(3-2x)$ $\left[-\frac{2x^3}{3-2x}+3x^2\ln(3-2x)\right]$

(17) $y=(x^2+1)[\ln(x^2+1)]^2$ $[4x\ln(x^2+1)+2x[\ln(x^2+1)]^2]$

(18) $y=\frac{x^2}{1+\ln x}$ $\left[\frac{x(1+2\ln x)}{(1+\ln x)^2}\right]$

(19) $y=\ln(x^2+4)$ $\left[\frac{2x}{x^2+4}\right]$

(20) $y=\ln(3x^2-7x^3)\sqrt{5x^2+8x}$ $\left[\frac{18x-21}{3x^2-7x}+\frac{10x+8}{10x^2+16x}\right]$

(21) $y = \ln(\log 3x^2)$ $[\frac{2}{x \ln 10 \log 3x^2}]$

(22) $y = \ln\left(\frac{2x}{1+x^2}\right)$ $[\frac{1-x^2}{x(1+x^2)}]$

(23) $y = \log\left(\ln \frac{x^3}{\sqrt{3x^4}}\right)$ $[\frac{1}{x \ln 10 \ln \frac{x^3}{\sqrt{3x^4}}}]$

(24) $y = \ln^7(3x^2 - 6x)$ $[\frac{7(6x-6)\ln^6(3x^2-6x)}{3x^2-6x}]$

(25) $y = \log\sqrt{\frac{8x^4 - 6x^3}{x^7}}$ $[\frac{(32x^3-18x^2)}{2(\ln 10)\cdot(8x^4-6x^3)} - \frac{7}{2x \ln 10}]$

(26) $y = \frac{\ln(x^2 - 3x)}{x^3}$ $[\frac{x^{-3}(2x-3)}{(x^2-3x)} - 3x^{-4}\ln(x^2-3x)]$

(27) $y = e^{e^{e^{2x}}}$ $[2e^{e^{e^{2x}}} e^{e^{2x}} e^{2x}]$

(28) $y = \ln\frac{e^{2x}+3}{3-2e^{7x}}$ $[\frac{2e^{2x}}{e^{2x}+3} + \frac{14e^{7x}}{3-2e^{7x}}]$

(29) $y = e^{2x}\log 3x^2$ $[\frac{2e^{2x}}{x \ln 10} + \log 3x^2 e^{2x} 2x]$

(30) $y = e^{\log_5 x}$ $[\frac{e^{\log_5 x}}{x \ln 5}]$

(31) $y = x^{2x} \cdot e^{-3x}$ $[-3x^{2x}e^{-3x} + 2x^{2x}e^{-3x} + 2x^{2x}\ln x]$

(32) $y = \ln\frac{1+e^{3x}}{1-e^{3x}}$ $[\frac{3e^{3x}}{1+e^{3x}} + \frac{3e^{3x}}{1-e^{3x}}]$

(33) $y = \ln(xe^{3x})$ $[\frac{1}{x} + 3]$

(34) $y = \sqrt{\ln(1-2x)}$ $[-\frac{1}{(1-2x)\sqrt{\ln(1-2x)}}]$

(35) $y = \frac{e^x}{\ln x}$ $[\frac{e^x(x\ln x - 1)}{x \ln^2 x}]$

(36) $y = (1+10^{\ln x})^3$ $[\frac{3(1+10^{\ln x})^2(10^{\ln x})\ln 10}{x}]$

2. จงหาอนุพันธ์ของฟังก์ชันต่อไปนี้ (ฟังก์ชันตรีโกณมิติและฟังก์ชันตรีโกณมิติผกผัน)

(1) $f(x) = \cos(3x^2+1)$ $[-6x\sin(3x^2+1)]$

(2) $f(x) = \sin 3x \cos 3x$ $[3\cos 6x]$

(3) $f(x) = 2\sin\sqrt{x}$ $[\frac{\cos\sqrt{x}}{\sqrt{x}}]$

(4) $f(x) = \tan^3\sqrt{x}$ $[\frac{3\tan^2\sqrt{x}\sec^2\sqrt{x}}{2\sqrt{x}}]$

(5) $f(x) = \frac{\sin x}{2+\cos x}$ $[\frac{2\cos x + 1}{(2+\cos x)^2}]$

(6) $f(x) = \frac{\operatorname{cosec} x}{\tan x}$ $[-\frac{\operatorname{cosec} x(1+\sec^2 x)}{\tan^2 x}]$

(7) $f(x)=\cos\sqrt{3x+7}$ $\left[\frac{-3\sin\sqrt{3x+7}}{2\sqrt{3x+7}}\right]$

(8) $f(x)=\tan(4x^2-x)$ $[(8x-1)\sec^2(4x^2-x)]$

(9) $f(x)=\sec\sqrt{x}$ $\left[\frac{\sec\sqrt{x}\tan\sqrt{x}}{2\sqrt{x}}\right]$

(10) $f(x)=\sin\frac{x}{x-2}$ $\left[\frac{-2\cos\frac{x}{x-2}}{(x-2)^2}\right]$

(11) $f(x)=\frac{1}{\tan x+\sin x}$ $\left[-\frac{\sec^2 x+\cos x}{(\tan x+\sin x)^2}\right]$

(12) $f(x)=\sqrt{1-\operatorname{cosec} x}$ $\left[\frac{\operatorname{cosec} x\cot x}{2\sqrt{1-\operatorname{cosec} x}}\right]$

(13) $f(x)=\sin^3\sqrt{2x+1}$ $\left[\frac{3\sin^2\sqrt{2x+1}\cos\sqrt{2x+1}}{\sqrt{2x+1}}\right]$

(14) $f(x)=\sqrt{\cos\sqrt{x}}$ $\left[\frac{-\sin\sqrt{x}}{4\sqrt{x\cos\sqrt{x}}}\right]$

(15) $f(x)=\operatorname{cosec}^{\frac{1}{3}}\frac{2}{x}$ $\left[\frac{2\operatorname{cosec}\frac{2}{x}\cot\frac{2}{x}}{3x^2\operatorname{cosec}^{\frac{2}{3}}\frac{2}{x}}\right]$

(16) $f(x)=\sec^2(x^2)$ $[4x\sec^2(x^2)\tan(x^2)]$

(17) $f(x)=\sin^4 2x+\cos^3 x$ $[8\sin^3 2x\cos 2x-3\sin x\cos^2 x]$

(18) $f(x)=\sqrt{x-\operatorname{cosec}^2 x}$ $\left[\frac{1+2\operatorname{cosec}^2 x\cot x}{2\sqrt{x-\operatorname{cosec}^2 x}}\right]$

(19) $f(x)=\frac{\sin\sqrt{2x}}{x}$ $\left[\frac{x\cos\sqrt{2x}-\sqrt{2x}\sin\sqrt{2x}}{\sqrt{2}x^{\frac{5}{2}}}\right]$

(20) $f(x)=\frac{x-1}{\cot\pi x}$ $\left[\frac{\cot\pi x+\pi(x-1)\operatorname{cosec}^2\pi x}{\cot^2\pi x}\right]$

(21) $f(x)=x^3\sin x-5\cos x$ $[x^3\cos x+(3x^2+5)\sin x]$

(22) $f(x)=\frac{\sec x}{1+\tan x}$ $\left[\frac{(1+\tan x)(\sec x\tan x)-\sec^3 x}{(1+\tan x)^2}\right]$

(23) $f(x)=x-4\operatorname{cosec} x+2\cot x$ $[1+4\operatorname{cosec} x\cot x-2\operatorname{cosec}^2 x]$

(24) $f(x)=\frac{\sin x\sec x}{1+x\tan x}$ $\left[\frac{1}{(1+x\tan x)^2}\right]$

(25) $f(x)=\frac{(x^2+1)\cot x}{2-\cos x\operatorname{cosec} x}$

$$\left[\frac{(2-\cos x\operatorname{cosec} x)[(x^2+1)(-\operatorname{cosec}^2 x)+2x\cot x]-(x^2+1)\cot x\operatorname{cosec} x(\cos x\cot x+\sin x)}{(2-\cos x\operatorname{cosec} x)^2}\right]$$

(26) $f(x)=\frac{\sin x+\cos x}{\cos x-\sin x}$ $\left[\frac{2(1+\sin 2x)}{\cos^2 2x}\right]$

(27) $f(x)=\sqrt{\dfrac{1-\cos x}{1+\cos x}}$ $\left[\dfrac{\sin x}{|\sin x|(1+\cos x)}\right]$

(28) $f(x)=2x\sin x-(x^2-2)\cos x$ $[x^2\sin x]$

(29) $f(x)=\sin^{-1}\dfrac{2x-1}{3}$ $\left[\dfrac{1}{\sqrt{2+x-x^2}}\right]$

(30) $f(x)=\cos^{-1}\dfrac{x+3}{5}$ $\left[-\dfrac{1}{\sqrt{16-6x-x^2}}\right]$

(31) $f(x)=(1+x^2)\tan^{-1}x$ $[2x\tan^{-1}x+1]$

(32) $f(x)=x^2\sec^{-1}(1+x^2)$ $\left[\dfrac{2x^3}{(1+x^2)\sqrt{2x+x^2}}+2x\sec^{-1}(1+x^2)\right]$

(33) $f(x)=\dfrac{\sin^{-1}x}{\sin^{-1}2x}$ $\left[\dfrac{\sqrt{1-4x^2}\sin^{-1}2x-2\sqrt{1-x^2}\sin^{-1}x}{\sqrt{1-x^2}\sqrt{1-4x^2}(\sin^{-1}2x)^2}\right]$

(34) $f(x)=(\sin^{-1}x^2)^{\frac{1}{2}}$ $\left[\dfrac{x}{\sqrt{(1-x^4)\sin^{-1}x^2}}\right]$

(35) $f(x)=\sqrt{a^2-x^2}+a\sin^{-1}\dfrac{x}{a};\ a>0$ $\left[\sqrt{\dfrac{a-x}{a+x}}\right]$

(36) $f(x)=\cos^{-1}\dfrac{a}{\sqrt{a^2+x^2}};\ a>0$ $\left[\dfrac{a}{\sqrt{a^2+x^2}}\right]$

(37) $f(x)=\csc^{-1}\sqrt{x}+\sec^{-1}\sqrt{x}$ $[0]$

(38) $f(x)=x\sqrt{1-x^2}-\cos^{-1}x$ $[2\sqrt{1-x^2}]$

(39) $f(x)=\sqrt{x^2-1}-\sec^{-1}x$ $\left[\dfrac{x|x|-1}{|x|\sqrt{x^2-1}}\right]$

(40) $f(x)=\cot^{-1}\sqrt{x-1}$ $\left[\dfrac{-1}{2x\sqrt{x-1}}\right]$

(41) $f(x)=\cot^{-1}\dfrac{1}{x}-\tan^{-1}x$ $[0]$

(42) $f(x)=x\sin^{-1}x+\sqrt{1-x^2}$ $[\sin^{-1}x]$

(43) $f(x)=\sin^{-1}(\cos x)$ $\left[\dfrac{-\sin x}{|\sin x|}\right]$

(44) $f(x)=\tan^{-1}(\csc x)$ $\left[\dfrac{-\csc x\cot x}{1+\csc^2 x}\right]$

(45) $f(x)=\sec^{-1}(\csc x)$ $\left[\dfrac{-\cos x}{|\cos x|}\right]$

(46) $f(x)=\sin(\sec^{-1}x)$ $\left[\dfrac{\cos(\sec^{-1}x)}{x\sqrt{x^2-1}}\right]$

(47) $f(x)=\tan(\cos^{-1}x)$ $\left[\dfrac{-\sec^2(\cos^{-1}x)}{\sqrt{1-x^2}}\right]$

(48) $f(x)=(\cot^{-1}x^2)^2$ $\left[\dfrac{-4x\cot^{-1}x^2}{1+x^4}\right]$

3. จงหาอนุพันธ์ของฟังก์ชันต่อไปนี้ (ฟังก์ชันไฮเพอร์โบลิกและฟังก์ชันไฮเพอร์โบลิกผกผัน)

(1) $f(x)=\sinh(2x+1)$ $[\,2\cosh(2x+1)\,]$

(2) $f(x)=\frac{1}{2}\operatorname{sech}2x$ $[\,-\operatorname{sech}2x\tanh 2x\,]$

(3) $f(x)=\frac{1}{2}\cot 2x-2\tanh\frac{x}{2}$ $[\,-\operatorname{csch}^2 2x-\operatorname{sech}^2\frac{x}{2}\,]$

(4) $f(x)=\frac{\cosh\sqrt{x}}{\sqrt{x}}$ $[\,\frac{\sqrt{x}\sinh\sqrt{x}-\cosh\sqrt{x}}{2x\sqrt{x}}\,]$

(5) $f(x)=\cosh^2 x+\sinh^2 x$ $[\,2\sinh 2x\,]$

(6) $f(x)=\tanh^3 2x$ $[\,6\tanh^2 2x\operatorname{sech}^2 2x\,]$

(7) $f(x)=\frac{\cosh x}{1-\sinh x}$ $[\,(1-\sinh x)^{-2}\,]$

(8) $f(x)=\sqrt{\operatorname{sech}x^2}$ $[\,-x\sqrt{\operatorname{sech}x^2}\tanh x^2\,]$

(9) $f(x)=\sinh x\cosh x$ $[\,\coth 2x\,]$

(10) $f(x)=\frac{\cosh x-\sinh x}{\cosh x+\sinh x}$ $[\,-2e^{-2x}\,]$

(11) $f(x)=(\sinh x+\cosh x)^4$ $[\,4e^{4x}\,]$

(12) $f(x)=(\tanh 2x)(1+\tanh^2 x)$ $[\,2\operatorname{sech}^2 x\,]$

(13) $f(x)=x\sinh x-\cosh x$ $[\,x\cosh x\,]$

(14) $f(x)=\tanh x-\frac{1}{3}\tanh^3 x$ $[\,\operatorname{sech}^4 x\,]$

(15) $f(x)=\tan^{-1}(\sinh x)$ $[\,\operatorname{sech}x\,]$

(16) $f(x)=\sin^{-1}(\tanh x)$ $[\,\operatorname{sech}x\,]$

(17) $f(x)=\frac{\sinh x}{1+\cosh x}$ $[\,(1+\cosh x)^{-1}\,]$

(18) $f(x)=\tanh(\cot x)$ $[\,-\operatorname{csch}^2 x\operatorname{sech}^2(\cot x)\,]$

(19) $f(x)=\sinh^2 3x-\cosh^2 3x$ $[0]$

(20) $f(x)=\sqrt{\operatorname{sech}x^2}-\operatorname{sech}^2\sqrt{x}$ $[\,-\frac{2x\operatorname{sech}x^2\tanh x^2}{\sqrt{\operatorname{sech}x}}+\frac{\operatorname{sech}^2\sqrt{x}\tanh\sqrt{x}}{\sqrt{x}}\,]$

(21) $f(x)=\sinh^{-1}2x$ $[\,\frac{2}{\sqrt{1+4x^2}}\,]$

(22) $f(x)=2\cosh^{-1}\sqrt{x}$ $[\,\frac{1}{\sqrt{x^2-x}}\,]$

(23) $f(x)=(1-x)\tanh^{-1}x$ $[\,\frac{1}{1+x}-\tanh^{-1}x\,]$

(24) $f(x)=(1-x^2)\coth^{-1}x$ $[\,1-2x\coth^{-1}x\,]$

(25) $f(x)=x\operatorname{sech}^{-1}x$ $[\,\frac{1}{-\sqrt{1-x^2}}+\operatorname{sech}^{-1}x\,]$

(26) $f(x)=x^2\operatorname{csch}^{-1}x^2$ $\left[\frac{-2x}{\sqrt{1+x^4}}+2x\operatorname{csch}^{-1}x^2\right]$

(27) $f(x)=\coth^{-1}\frac{x-1}{x+1}$ $\left[\frac{1}{2x}\right]$

(28) $f(x)=\operatorname{sech}^{-1}\sqrt{x+1};\ -1<x<0$ $\left[-\frac{1}{2(x+1)\sqrt{-x}}\right]$

(29) $f(x)=(\sinh^{-1}2x)^3$ $\left[\frac{6(\sinh^{-1}2x)^2}{\sqrt{1+4x^2}}\right]$

(30) $f(x)=\cosh^{-1}(\cosh x)$ $[1]$

(31) $f(x)=\operatorname{csch}^{-1}(\tan x)$ $[-\csc x]$

(32) $f(x)=\frac{1}{\sinh^{-1}x}$ $\left[\frac{-1}{(\sinh^{-1}x)^2\sqrt{x^2+1}}\right]$

(33) $f(x)=x\sinh^{-1}x-\sqrt{x^2+1}$ $[\sinh^{-1}x]$

(34) $f(x)=\frac{x^2}{2}\operatorname{sech}^{-1}x-\frac{1}{2}\sqrt{1-x^2}$ $[x\operatorname{sech}^{-1}x]$

(35) $f(x)=\frac{(x^2-1)}{2}\coth^{-1}x+\frac{x}{2}$ $[x\coth^{-1}x]$

(36) $f(x)=\cosh^{-1}(2x^2+1)$ $\left[\frac{2}{\sqrt{x^2+1}}\right]$

(37) $f(x)=\tanh^{-1}(x^2+1)$ $\left[-\frac{2}{x(x^2+2)}\right]$

(38) $f(x)=\sinh x\coth^{-1}x$ $\left[\frac{\sinh x}{\sqrt{x^2-1}}+\cosh x\cosh^{-1}x\right]$

(39) $f(x)=(1+x\operatorname{csch}^{-1}x)^{10}$ $\left[10(1+x\operatorname{csch}^{-1}x)^9\left(\operatorname{csch}^{-1}x-\frac{x}{|x|\sqrt{1+x^2}}\right)\right]$

(40) $f(x)=\sinh(\cosh^{-1}3x)+\sinh^{-1}(\cosh x^3)$ $\left[\frac{3\cosh(\cosh^{-1}3x)}{\sqrt{9x^2-1}}+\frac{3x^2\sinh x^3}{\sqrt{\cosh(\cosh^{-1}3x)}}\right]$

(41) $f(x)=\frac{2x^2-1}{4}\cosh^{-1}x-\frac{x\sqrt{x^2-1}}{4}$ $[x\cosh^{-1}x]$

(42) $f(x)=(\sinh^{-1}\sqrt{x})^2+(\cosh^{-1}\sqrt{x})^2$ $\left[\frac{1}{\sqrt{x}}\left(\frac{\sinh^{-1}\sqrt{x}}{\sqrt{x+1}}+\frac{\cosh^{-1}\sqrt{x}}{\sqrt{x-1}}\right)\right]$

4. จงหาอนุพันธ์ของฟังก์ชันต่อไปนี้ (การหาอนุพันธ์โดยอาศัยลอการิทึมธรรมชาติ ln)

(1) $y=\frac{x\sqrt{1-x}}{(x+2)(x-3)}$ $\left[\frac{x\sqrt{1-x}}{(x+2)(x-3)}\left[\frac{1}{x}-\frac{1}{2(1-x)}-\frac{1}{x+2}-\frac{1}{x-3}\right]\right]$

(2) $y=\frac{\sqrt[3]{(x+1)^2}}{x^4(3-4x)^2}$ $\left[\frac{\sqrt[3]{(x+1)^2}}{x^4(3-4x)^2}\left[\frac{2}{3(x+1)}-\frac{4}{x}+\frac{8}{3-4x}\right]\right]$

(3) $y=\frac{(x-1)^2\sqrt{3x+4}}{x+2}$ $\left[\frac{(x-1)^2\sqrt{3x+4}}{x+2}\left[\frac{2}{x-1}+\frac{3}{2(3x+4)}-\frac{1}{x+2}\right]\right]$

(4) $y=\sqrt{\frac{(x-1)(x+2)}{(x-3)^3}}$ $\left[\frac{1}{2}\sqrt{\frac{(x-1)(x+2)}{(x-3)^2}}\left[\frac{1}{x-1}+\frac{1}{x+2}-\frac{3}{x-3}\right]\right]$

(5) $y=(x^2+1)^3(x^3-1)^5(x^5+1)^7$ $\left[(x^2+1)^3(x^3-1)^5(x^5+1)^7\left[\frac{6x}{x^2+1}+\frac{15x^2}{x^3-1}+\frac{35x^4}{x^5+1}\right]\right]$

(6) $y=\frac{(2x+3)^2(x-4)^3}{(7x-2)^4(x^2+2x)^5}$ $\left[\frac{(2x+3)^2(x-4)^3}{(7x-2)^4(x^2+2x)^5}\left[\frac{4}{2x+3}+\frac{3}{x-4}-\frac{28}{7x-2}-\frac{10x+10}{x^2+2x}\right]\right]$

(7) $y=x^3e^{-2x}\ln x$ $\left[x^2e^{-2x}(3-2x)\ln(x+1)\right]$

(8) $y=\frac{e^x\sin 2x}{x}$ $\left[x^{-2}e^x\left[(x-1)\sin 2x+2x\cos 2x\right]\right]$

(9) $y=\frac{\sin x\cos 2x}{\sec x}$ $\left[\cos 4x\right]$

(10) $y=\sqrt{\frac{\sin 3x}{1-\sin 3x}}$ $\left[\frac{3}{2}\sqrt{\frac{\sin 3x}{1-\sin 3x}}\left(\frac{3\cot 3x}{1-\sin 3x}\right)\right]$

(11) $y=\frac{e^x(1+x)^2}{\sqrt{(1-x)^3}}$ $\left[\frac{e^x(1+x)(9-x-2x^2)}{2\sqrt{(1-x)^5}}\right]$

(12) $y=\frac{x^3\sqrt{2x+1}}{(x-3)^2}$ $\left[\frac{x^3\sqrt{2x+1}}{(x-3)^2}\left[\frac{3}{x}+\frac{1}{2x+1}-\frac{2}{x-3}\right]\right]$

(13) $y=\frac{(x-1)^5(x^2+1)^{\frac{2}{3}}}{\sqrt{x^3-5x^2+6x}}$ $\left[\frac{(x-1)^5(x^2+1)^{\frac{2}{3}}}{\sqrt{x^3-5x^2+6x}}\left[\frac{5}{x-1}+\frac{4x}{3(x^2+1)}-\frac{3x^2-10x+6}{2(x^3-5x^2+6x)}\right]\right]$

(14) $y=\frac{x^2e^x}{2\sqrt{x+1}}$ $\left[\frac{x^2e^x}{2\sqrt{x+1}}\left[\frac{2}{x}+1-\frac{1}{2(x+1)}\right]\right]$

(15) $y=\frac{\sin x\cos x}{e^x2^x}$ $\left[\frac{\sin x\cos x}{e^x2^x}\left[\cot x-\tan x-1-\ln 2\right]\right]$

(16) $y=\frac{\cos x}{\sqrt{4-3x^2}}$ $\left[\frac{\cos x}{\sqrt{4-3x^2}}\left[\frac{3x}{4-3x^2}-\tan x\right]\right]$

(17) $y=\sqrt{x}\sqrt[3]{x+3}\sqrt[5]{3x-2}$ $\left[\sqrt{x}\sqrt[3]{x+3}\sqrt[5]{3x-2}\left[\frac{1}{2x}+\frac{1}{3(x+3)}+\frac{3}{5(3x-2)}\right]\right]$

(18) $y=x\sqrt[3]{1+x^2}$ $\left[x\sqrt[3]{1+x^2}\left[\frac{1}{x}+\frac{2x}{3(1+x)^2}\right]\right]$

(19) $y=\frac{(x^2-8)^{\frac{1}{3}}\sqrt{x^3+1}}{x^6-7x+5}$ $\left[\frac{(x^2-8)^{\frac{1}{3}}\sqrt{x^3+1}}{x^6-7x+5}\left[\frac{2x}{3(x^2-8)}+\frac{3x^2}{2(x^3+1)}-\frac{6x^5-7}{x^6-7x+5}\right]\right]$

(20) $y=\sqrt{\frac{2x-1}{x^2+1}}$ $\left[\sqrt{\frac{2x-1}{x^2+1}}\left[\frac{1}{2x-1}-\frac{x}{x^2+1}\right]\right]$

(21) $y=\frac{\sqrt{x}\sqrt[3]{x+1}}{\sin x\sec x}$ $\left[\frac{\sqrt{x}\sqrt[3]{x+1}}{\sin x\sec x}\left[\frac{1}{2x}+\frac{1}{3(x+1)}-\cot x-\tan x\right]\right]$

(22) $y=\sqrt[5]{\frac{1-x^3}{1+3x^2}}$ $\left[\frac{1}{5}\sqrt[5]{\frac{1-x^3}{1+3x^2}}\left[\frac{3x^2}{1-x^3}-\frac{6x}{1+3x^2}\right]\right]$

(23) $y=\frac{\sin x\cos^2 x\tan^3 x}{\sqrt{x}}$ $\left[\frac{\sin x\cos^2 x\tan^3 x}{\sqrt{x}}\left[\cot x-2\tan x+\frac{3\sec^2 x}{\tan x}-\frac{1}{2x}\right]\right]$

(24) $y=(\cos x)^x-x^{\cos x}$ $\left[(\cos x)^x(\ln(\cos x)-x\tan x)+x^{\cos x}\left[\frac{\cos x}{x}-(\ln x)\sin x\right]\right]$

(25) $y=\dfrac{\sqrt[3]{x^2}(1-x)\sin^3 x\cos^2 x}{1+x^2}$ $\left[\dfrac{\sqrt[3]{x^2}(1-x)\sin^3 x\cos^2 x}{1+x^2}\left(\dfrac{2}{3x}-\dfrac{1}{1-x}+3\cot x-2\tan x+\dfrac{2x}{1-x^2}\right)\right]$

(26) $y=\dfrac{e^{2x}(2x-1)^6}{(x^3+5)^2(4-7x)}$ $\left[\dfrac{e^{2x}(2x-1)^6}{(x^3+5)^2(4-7x)}\left(2+\dfrac{12}{2x-1}-\dfrac{6x^2}{x^3+5}+\dfrac{7}{4-7x}\right)\right]$

5. จงหาอนุพันธ์ของฟังก์ชันต่อไปนี้

(1) $f(x)=\ln(\sin 5x)$ $[5\cot 5x]$

(2) $f(x)=\ln\left(\cot\dfrac{x}{2}\right)$ $[-\operatorname{cosec} x]$

(3) $f(x)=(\sin x^2)^{4x}$ $[(\sin x^2)^{4x}[8x^2\cot x^2+4\ln(\sin x^2)]]$

(4) $f(x)=3^x\sec x$ $[(3^x\sec x)(\ln 3+\tan x)]$

(5) $f(x)=x\ln(\operatorname{cosec}^2 x)$ $[-2x\cot x+\ln(\operatorname{cosec}^2 x)]$

(6) $f(x)=\sec x\operatorname{cosec} x\ln(\cot x)$ $[\sec x\operatorname{cosec} x(\tan x\ln\cot x-\cot x\ln\cot x-\tan x\operatorname{cosec}^2 x)]$

(7) $f(x)=e^x\cot^{-1}\sqrt{x}$ $\left[-\dfrac{e^x}{2(1+x)\sqrt{x}}+e^x\cot^{-1}\sqrt{x}\right]$

(8) $f(x)=a^x\arcsin 2x$ $\left[\dfrac{2a^x}{\sqrt{1-4x^2}}+(a^x\ln a)\arcsin 2x\right]$

(9) $f(x)=\operatorname{arccosec}(\ln x^2)$ $\left[-\dfrac{2}{x|\ln x^2|\sqrt{(\ln x^2)^2-1}}\right]$

(10) $f(x)=\tan(\arcsin\sqrt{x})$ $\left[\dfrac{\sec^2(\arcsin\sqrt{x})}{2\sqrt{x(1-x)}}\right]$

(11) $f(x)=e^x\operatorname{csch} x^2$ $[e^x\operatorname{csch} x^2(1-2x\coth x^2)]$

(12) $f(x)=\sqrt{\cosh(\ln x)}$ $\left[\dfrac{\sinh(\ln x)}{2x\sqrt{\cosh(\ln x)}}\right]$

(13) $f(x)=x^{\sinh x}$ $\left[x^{\sinh x}\left(\dfrac{\sinh x}{x}+\ln x\cosh x\right)\right]$

(14) $f(x)=\ln\left(\dfrac{1+\sin x}{1-\sin x}\right)$ $\left[\dfrac{2}{\cos x}\right]$

(15) $f(x)=x\tan\dfrac{x}{2}+2\ln\left(\cos\dfrac{x}{2}\right)$ $\left[\dfrac{x}{1+\cos x}\right]$

(16) $f(x)=x+\ln|\sin x-\cos x|$ $\left[\dfrac{2\tan x}{\tan x-1}\right]$

(17) $f(x)=\ln\left|\tan\left(\dfrac{\pi}{4}+\dfrac{x}{2}\right)\right|-\operatorname{cosec} x$ $[\operatorname{cosec}^2 x\sec x]$

(18) $f(x)=\ln x-\dfrac{1}{2}\ln(1+x^2)-\dfrac{\arctan x}{x}$ $\left[\dfrac{\arctan x}{x^2}\right]$

(19) $f(x)=x\ln(1+x^2)-2x+2\arctan x$ $[\ln(1+x^2)]$

(20) $f(x)=x\tan^{-1}\dfrac{x}{a}-\dfrac{a}{2}\log_e(x^2+a^2);\ a>0$ $\left[\arctan\dfrac{x}{a}\right]$