

แบบฝึกหัดชุดที่ 4

อนุพันธ์อันดับสูงและอนุพันธ์ของฟังก์ชันโดยปริยาย

1. จงหาอนุพันธ์อันดับที่สองของฟังก์ชันต่อไปนี้

(1) $f(x) = 3x^4 - 2x^3 + x^2 - 4x + 2$ [$36x^2 - 12x + 2$]

(2) $f(x) = x^2 + \frac{1}{x^2}$ [$2 + \frac{6}{x^4}$]

(3) $f(x) = \sqrt{x^2 + 4}$ [$\frac{4}{\sqrt{(x+4)^3}}$]

(4) $f(x) = \frac{a+bx}{a-bx}$ [$\frac{4ab^2}{(a-bx)^3}$]

(5) $f(x) = 3x^4 - 2x^3 + 6x$ [$36x^2 - 12x$]

(6) $f(x) = \sqrt{a+bx}$ [$\frac{-b^2}{4\sqrt{(a+bx)^3}}$]

(7) $f(x) = \frac{x^2}{5+x}$ [$\frac{50}{(5+x)^3}$]

(8) $f(x) = x^3 - \frac{3}{x}$ [$6x - \frac{6}{x^3}$]

(9) $f(x) = \sqrt{25-3x}$ [$\frac{-9}{4\sqrt{(25-3x)^3}}$]

(10) $f(x) = \sqrt{ax} + \frac{a^2}{\sqrt{ax}}$ [$\frac{-\sqrt{a}}{4\sqrt{x^3}} + \frac{3a^2}{4\sqrt{ax^5}}$]

(11) $f(x) = x\sqrt{3x-2}$ [$\frac{18x-4}{\sqrt{12x^3-8x^2}} - \frac{2(9x^2-4x)^2}{\sqrt{(12x^3-8x^2)^3}}$]

(12) $f(x) = \frac{x^2}{x^2+7}$ [$\frac{14}{(x^2+7)^2} - \frac{56x^2}{(x^2+7)^3}$]

(13) $f(x) = \sin 3x$ [$-9 \sin 3x$]

(14) $f(x) = e^x \cos x$ [$-2e^x \sin x$]

(15) $f(x) = (x^2+1)^5$ [$10(x^2+1)^4 + 80x^2(x^2+1)^3$]

(16) $f(x) = \sqrt{4x^3+2}$ [$\frac{12x^4+24x}{\sqrt{(4x^3+2)^3}}$]

(17) $f(x) = \frac{1}{\sqrt{7x^2-x}}$ [$\frac{392x^2-56x+3}{4\sqrt{(7x^2-x)^5}}$]

(18) $f(x) = \frac{x}{2x-3}$ [$\frac{12}{(2x-3)^3}$]

(19) $f(x) = \frac{x^2}{1-2x}$ [$\frac{2}{(1-2x)^3}$]

$$(20) \quad f(x) = \sqrt[3]{2x^3 - 5} \quad \left[\frac{-20x}{(2x^3 - 5)^{\frac{5}{3}}} \right]$$

2. จงหาอนุพันธ์อันดับที่ n ของฟังก์ชันต่อไปนี้

$$(1) \quad f(x) = x^4 - 2x^3 + x^2 - 5x + 4; \quad n = 3 \quad [24x - 12]$$

$$(2) \quad f(x) = \ln(2^{5x} \cdot 3^{4x^2}); \quad n = 2 \quad [8 \ln 3]$$

$$(3) \quad f(x) = x|x| + 2|x|; \quad n = 2 \quad [f'(x) = \begin{cases} 2, & x \geq 0 \\ -2, & x < 0 \end{cases}]$$

$$(4) \quad f(x) = x\sqrt{x-2}; \quad n = 3 \quad \left[\frac{12-3x}{8(x-2)^{\frac{5}{2}}} \right]$$

$$(5) \quad f(x) = x^{\frac{7}{2}} - 2x^{\frac{5}{2}} + x^{\frac{1}{2}}; \quad n = 4 \quad \left[\frac{105}{16x^{\frac{1}{2}}} + \frac{15}{8x^{\frac{3}{2}}} - \frac{15}{16x^{\frac{7}{2}}} \right]$$

$$(6) \quad f(x) = x^5 - 2x^3 + 3x^2 - 4; \quad n = 3 \quad [60x^2 - 12]$$

$$(7) \quad f(x) = \sqrt{2x - x^2}; \quad n = 3 \quad \left[\frac{3(1-x)}{(2x-x^2)^{\frac{5}{2}}} \right]$$

$$(8) \quad f(x) = x^k; \quad k \in I^+, \quad n = n \quad [n!]$$

$$(9) \quad f(x) = (1-x)^k; \quad k \in I^+, \quad n = n \quad [(-1)^n n!]$$

$$(10) \quad f(x) = \frac{1}{1-2x}; \quad n = n \quad \left[\frac{2^n n!}{(1-2x)^{n+1}} \right]$$

3. จงหา $f''(a)$ เมื่อกำหนดฟังก์ชัน f และจำนวนจริง a ดังต่อไปนี้

$$(1) \quad f(x) = x^4 - 7x^3 + 2x^2 + 5; \quad a = 1 \quad [-26]$$

$$(2) \quad f(x) = 5x^3 - 3x^5; \quad a = -1 \quad [30]$$

$$(3) \quad f(x) = 4x^2 - 8x + 1; \quad a = 2 \quad [8]$$

$$(4) \quad f(x) = \frac{x^4}{4} - \frac{x^3}{3} + \frac{x^2}{2} - x + 3; \quad a = -2 \quad [17]$$

$$(5) \quad f(x) = 2x^4 - 4x^2 - 8; \quad a = \frac{1}{2} \quad [-2]$$

$$(6) \quad f(x) = \frac{1}{2}x^4 - \frac{3}{2}x^2 - x; \quad a = -\frac{1}{2} \quad \left[-\frac{3}{2} \right]$$

$$(7) \quad f(x) = 3x^7 - 7x^3 + 21x^2; \quad a = 0 \quad [42]$$

$$(8) \quad f(x) = x^2(x^5 - 1); \quad a = 1 \quad [40]$$

$$(9) \quad f(x) = (x-2)(x-3); \quad a = -1 \quad [2]$$

$$(10) \quad f(x) = (3x-1)(2x+5); \quad a = 2 \quad [12]$$

4. จงหา $f^{(n)}(a)$ เมื่อกำหนดฟังก์ชัน f จำนวนเต็มบวก n และจำนวนจริง a ดังต่อไปนี้

$$(1) \quad f(x) = 2x - \frac{1}{x}; \quad n = 3, \quad a = 1 \quad [6]$$

$$(2) \quad f(x) = x^3 - \frac{1}{x^3}; \quad n = 3, \quad a = 1 \quad [66]$$

- (3) $f(x) = \frac{x}{x+1}; n=4, a=-2$ [24]
- (4) $f(x) = \frac{x}{x^2-1}; n=3, a=0$ [-6]
- (5) $f(x) = \sqrt{x^2+1}; n=2, a=\sqrt{3}$ [$\frac{1}{8}$]
- (6) $f(x) = \frac{x-3}{2x+1}; n=3, a=-\frac{1}{4}$ [2688]
- (7) $f(x) = (4x+7)^5; n=5, a=-2$ [122880]
- (8) $f(x) = \ln(3x+1); n=3, a=-1$ [$-\frac{27}{4}$]
- (9) $f(x) = \sin x; n=6, a=\frac{\pi}{4}$ [$-\frac{1}{\sqrt{2}}$]
- (10) $f(x) = \frac{\tan \frac{x}{2}}{\sec 2x}; n=3, a=\frac{\pi}{2}$ [10]
- (11) $f(x) = \arcsin[\cos(\pi+x)]; n=2, a=-\frac{3\pi}{2}$ [0]

5. สมมติว่าสมการต่อไปนี้กำหนดฟังก์ชันโดยปริยายที่มีอนุพันธ์ จงหาอนุพันธ์โดยปริยาย ($\frac{dy}{dx}$)

- (1) $2y^2 = x^3$ [$\frac{3x^2}{4y}$]
- (2) $xy^3 = 12$ [$-\frac{y}{3x}$]
- (3) $y^2 + 6x^2 = 5$ [$-\frac{6x}{y}$]
- (4) $x^3 + 3x^2y + 7y + 1 = 0$ [$-\frac{3x(x+2y)}{3x^2+7}$]
- (5) $y^2 + 6xy - 5x + 7 = 0$ [$\frac{5-6y}{2y+6x}$]
- (6) $\sqrt{x} + \sqrt{y} = 4$ [$-\sqrt{\frac{y}{x}}$]
- (7) $\sqrt{xy} + 3y = 10x$ [$\frac{20\sqrt{xy}-y}{x+6\sqrt{xy}}$]
- (8) $6x - \sqrt{2xy} + xy^3 = y^2$ [$\frac{(y^3+6)\sqrt{2xy}-y}{y(2-3xy)\sqrt{2xy+x}}$]
- (9) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$ [$-\left(\frac{y}{x}\right)^{\frac{1}{3}}$]
- (10) $\frac{y^2}{x^3} = 1 + y^{\frac{3}{2}}$ [$\frac{6y^2}{4xy-3x^4\sqrt{y}}$]
- (11) $xy^2 + \sqrt{xy} = 2$ [$\frac{-y(2y\sqrt{xy}+1)}{x(4y\sqrt{xy}+1)}$]

- (12) $(x+2y)^2 + 2xy^2 = 6$ $\left[-\frac{x+2y+y^2}{2(x+2y+xy)} \right]$
- (13) $4x^2 + 9y^2 = 36$ $\left[-\frac{4x}{9y} \right]$
- (14) $xy^2 - x + 16 = 0$ $\left[\frac{1-y^2}{2xy} \right]$
- (15) $x^3 - 3x^2y + 19xy = 0$ $\left[\frac{3x^2 - 6xy + 19y}{3x^2 - 19x} \right]$
- (16) $x^2y + 3xy^2 - x = 3$ $\left[\frac{1-2xy-3y^2}{x^2-9xy^2} \right]$
- (17) $\frac{1}{y} + \frac{1}{x} = 2$ $\left[-\frac{y^2}{x^2} \right]$
- (18) $(x^2 + 3y^2)^{35} = x$ $\left[\frac{1-70x(x^2+3y^2)^{34}}{210y(x^2+3y^2)^{34}} \right]$
- (19) $3xy = (x^3 + y^2)^{\frac{3}{2}}$ $\left[\frac{\frac{3}{2}x^2(x^3+y^2)^{\frac{1}{2}} - y}{x - y(x^3+y^2)^{\frac{1}{2}}} \right]$
- (20) $xy^{\frac{2}{3}} + yx^{\frac{2}{3}} = x^2$ $\left[\frac{6x-3y-2x^{\frac{1}{3}}y}{2xy^{\frac{1}{3}}+3x^{\frac{2}{3}}} \right]$
- (21) $\sin(x^2y^2) = x$ $\left[\frac{\sec(x^2y^2) - 2xy^2}{2x^2y} \right]$
- (22) $x^2 = \frac{\cot y}{1 + \operatorname{cosec} y}$ $\left[\frac{2x(1 + \operatorname{cosec} y)^2}{[(1 + \operatorname{cosec} y)(-\operatorname{cosec}^2 y) + (\operatorname{cosec} y \cot^2 y)]} \right]$
- (23) $\tan^3(xy^2 + y) = x$ $\left[\frac{\cot^2(xy^2 + y) - 3y^2}{3(2xy + 1)} \right]$
- (24) $\frac{xy^3}{1 + \sec y} = 1 + y^4$ $\left[\frac{(xy^3)(\sec y \tan y) - y^3(1 + \sec y)}{3xy^2(1 + \sec y) - 4y^3(1 + \sec y)^2} \right]$
- (25) $\sqrt{1 + \sin^3(xy^2)} = y$ $\left[\frac{3y^2 \sin^2(xy^2) \cos(xy^2)}{2\sqrt{1 + \sin^3(xy^2)} - 6xy \sin^2(xy^2) \cos(xy^2)} \right]$

6. สมมติว่าสมการต่อไปนี้กำหนดฟังก์ชันโดยปริยายที่มีอนุพันธ์ จงหาอนุพันธ์โดยปริยาย $\left(\frac{dy}{dx}\right)$ ณ จุด

$P(x, y)$ ที่กำหนดให้

- (1) $(y-x)^2 = 2x+4$; $P(6, 2)$ $\left[\frac{3}{4} \right]$
- (2) $x^2 + xy - y^2 = 1$; $P(2, 3)$ $\left[\frac{7}{4} \right]$
- (3) $x^2 + y^2 = 25$; $P(3, -4)$ $\left[\frac{3}{4} \right]$
- (4) $x^2y^2 = 9$; $P(-1, 3)$ $[3]$
- (5) $\frac{x-y}{x-2y} = 2$; $P(3, 1)$ $\left[\frac{1}{3} \right]$

- (6) $2x^3 - x^2y + y^3 = 1; P(2, -3)$ $[-\frac{36}{23}]$
- (7) $x^3 + 2xy = 5; P(1, 2)$ $[-\frac{7}{2}]$
- (8) $x^2 + \frac{x}{y} = -2; P(1, -\frac{1}{3})$ $[-\frac{1}{9}]$
- (9) $(\sqrt{x} + 1)(\sqrt{y} + 2) = 8; P(1, 4)$ $[-4]$
- (10) $y^2 + 3\sqrt{xy} = 27 + x^2; P(1, 4)$ $[-\frac{4}{11}]$
- (11) $2x^4 - 3x^2y^2 + y^4 = 0; P(-1, 1)$ $[-1]$
- (12) $(x + y)^2 - (x - y)^2 = x^4 + y^4; P(\sqrt{2}, \sqrt{2})$ $[-1]$
- (13) $y = 3xy^2; P(3, \frac{1}{9})$ $[-\frac{1}{27}]$
- (14) $x^2 + xy + y^2 - 3 = 0; P(2, 1)$ $[-\frac{5}{4}]$
- (15) $y^4 - xy^3 + x^2 - 7 = 0; P(3, 1)$ $[\frac{5}{2}]$
- (16) $y^5 + 3x^2y^3 - 7x^6 - 6 = 0; P(2, 1)$ $[\frac{1332}{41}]$
- (17) $\sqrt{x} + \sqrt{y} + \sqrt{xy} = 19; P(16, 9)$ $[-\frac{3}{5}]$
- (18) $y = x \sin\left(\frac{\pi}{6}(x^2 + 2y)\right); P(1, 1)$ $[1]$
- (19) $\sqrt{x} + \sqrt{\sqrt{x} + \cos y} = 2; P(1, \frac{\pi}{2})$ $[\frac{3}{2}]$
- (20) $2xy + \pi \sin y = 2\pi; P(1, \frac{\pi}{2})$ $[-\frac{\pi}{2}]$
- (21) $x \sin 2y = y \cos 2x; P(\frac{\pi}{4}, \frac{\pi}{2})$ $[2]$
- (22) $xe^y + y - 2x = \ln 2; P(1, \ln 2)$ $[0]$
- (23) $\ln x = 1 - \ln y; P(e, 1)$ $[\frac{e-1}{e}]$

7. จงใช้วิธีการหาอนุพันธ์โดยปริยายหา $\frac{d^2y}{dx^2}$ ของฟังก์ชันต่อไปนี้

- (1) $x^3 + y^3 = 1$ $[-\frac{2x}{y^5}]$
- (2) $2xy - y^2 = 3$ $[-\frac{3}{(y-x)^3}]$
- (3) $x^2 + y^2 = 1$ $[\frac{x^2 - y^2}{y^3}]$
- (4) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$ $[\frac{\frac{1}{4}y^{\frac{1}{3}}}{3x^{\frac{2}{3}}} + \frac{1}{3y^{\frac{1}{3}}x^{\frac{2}{3}}}]$

- (5) $y^2 = x^2 + 2x$ [$\frac{y^2 - (x+1)^2}{y^3}$]
- (6) $y^2 + 2y = 2x + 1$ [$-\frac{1}{(y+1)^3}$]
- (7) $x \cos y = y$ [$-\frac{\sin 2y + y(\sin^2 y + 1)}{(1 + x \sin y)^3}$]
- (8) $y^3 - y^2 = 4$ [$\frac{6y^3 - 8x^2}{9y^5}$]
- (9) $y^2 + 2xy = 16$ [$\frac{16}{(x+y)^3}$]
- (10) $y^2 + xy - x^2 = 9$ [$\frac{90}{(2y+x)^3}$]

8. จงใช้วิธีการหาอนุพันธ์โดยปริยาย หา $\frac{d^2y}{dx^2}$ ที่จุด $P(x,y)$ ที่กำหนดให้ต่อไปนี้

- (1) $x^2 - 4y^2 = 9$; $P(5,2)$ [$-\frac{9}{128}$]
- (2) $\cos(x+2y) = 0$; $P(\frac{\pi}{6}, \frac{\pi}{6})$ [0]
- (3) $xy + y^2 = 1$; $P(0,-1)$ [$-\frac{1}{4}$]
- (4) $x^3 + y^3 = 16$; $P(2,2)$ [-2]
- (5) $x^2 - xy + y^2 = 1$; $P(-1,-1)$ [6]
- (6) $y^3 + y - x = 1$; $P(1,1)$ [$-\frac{3}{32}$]
- (7) $y^3 - x^2 = 4$; $P(2,2)$ [$\frac{1}{18}$]
- (8) $y^3 + y = x^2 + 1$; $P(3,2)$ [$-\frac{94}{2197}$]
- (9) $xy + yx^2 = 2$; $P(1,1)$ [$\frac{7}{2}$]
- (10) $x \cos y = y$; $P(0,0)$ [0]

9. ถ้า $y = 3e^{2x} \cos(2x-3)$ แล้ว $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 8y$ มีค่าเท่าไร [0]

10. ถ้า $y = (\arcsin x)^2$ แล้ว

- (1) $(1-x^2)\left(\frac{dy}{dx}\right)^2$ มีค่าเท่าไร [4y]
- (2) $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2$ มีค่าเท่าไร [0]

11. ถ้า $y = \sin(2\sin^{-1} x)$ แล้ว $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y$ มีค่าเท่าไร [0]

12. ถ้า $y = \sqrt{1-x^2} \arcsin x$ แล้ว $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx}$ มีค่าเท่าไร [-2x - y]