

Worksheet: Integration using Partial Fractions

1. Evaluate the following indefinite integrals.

$$(1) \int \frac{1}{2x^3 + x^2 - x} dx \qquad (2) \int \frac{3x^3 - 5x^2 - 11x + 9}{x^2 - 2x - 3} dx$$

$$(3) \int \frac{x^2 + 12x - 5}{(x+1)^2(x-7)} dx \qquad (4) \int \frac{8x^2 - 3x - 4}{(4x-1)(x^2+1)} dx$$

$$(5) \int \frac{4x^3 + 2x^2 + 1}{4x^3 - x} dx \qquad (6) \int \frac{3x - 2}{x^3 + x^2 - x - 1} dx$$

$$(7) \int \frac{6x^2 - x - 1}{3x - 1} dx \qquad (8) \int \frac{3x + 5}{x^2 + 4x + 13} dx$$

$$(9) \int \frac{1}{x^2 - 4} dx \qquad (10) \int \frac{2x + 3}{x^2 - 9} dx$$

$$(11) \int \frac{2 - x}{x^2 + 5x} dx \qquad (12) \int \frac{x^2 - 1}{x^2 - 16} dx$$

$$(13) \int \frac{x^4 + x^3 + x^2 + 1}{x^2 + x - 2} dx \qquad (14) \int \frac{x^2 + x - 1}{x(x^2 - 1)} dx$$

$$(15) \int \frac{x + 7}{x^2(x + 2)} dx \qquad (16) \int \frac{x^5 + 1}{x^3(x + 2)} dx$$

$$(17) \int \frac{2x - 1}{x^2 - x - 6} dx \qquad (18) \int \frac{3x + 11}{x^2 - x - 6} dx$$

ANSWERS

1. $\frac{2}{3} \ln |2x - 1| + \frac{1}{3} \ln |x + 1| - \ln |x| + C$
2. $\frac{3}{2}x^2 + x + 3 \ln |x - 3| - 3 \ln |x + 1| + C$
3. $2 \ln |x - 7| - \ln |x + 1| - \frac{2}{x+1} + C$
4. $\frac{3}{2} \ln |x^2 + 1| - \ln |4x - 1| + C$
5. $x + \ln \frac{|2x-1|}{|x|} + \frac{1}{2} \ln |2x + 1| + C$
6. $\frac{1}{4} \ln \frac{|x-1|}{|x+1|} - \frac{5}{2(x+1)} + C$
7. $x^2 + \frac{1}{3}x - \frac{2}{9} \ln |3x - 1| + C$
8. $\frac{3}{2} \ln(x^2 + 4x + 13) - \frac{1}{3} \arctan(\frac{1}{3}(x + 2)) + C$
9. $\frac{1}{4} \ln \frac{|x-2|}{|x+2|} + C$
10. $\frac{1}{2} \ln |x + 3| + \frac{3}{2} \ln |x - 3| + C$
11. $\frac{2}{5} \ln |x| - \frac{7}{5} \ln |x + 5| + C$
12. $x + \frac{15}{8} \ln \frac{|x-4|}{|x+4|} + C$
13. $\frac{x^3}{3} + 3x - \frac{13}{3} \ln |x + 2| + \frac{4}{3} \ln |x - 1| + C$
14. $\ln |x| + \frac{1}{2} \ln \frac{|x-1|}{|x+1|} + C$
15. $-\frac{5}{4} \ln |x| - \frac{7}{2x} + \frac{5}{4} \ln |x + 2| + C$
16. $\frac{x^2}{2} - 2x + \frac{1}{8} \ln |x| + \frac{1}{4x} - \frac{1}{4x^2} + \frac{31}{4} \ln |x + 2| + C$
17. $\ln |x^2 - x - 6| + C$
18. $4 \ln |x - 3| - \ln |x + 2| + C$