

Worksheet: Integration by Substitution

1. Evaluate the following indefinite integrals.

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| (1) $\int \frac{x^2}{(1+x^3)^2} dx$ | (2) $\int 10x\sqrt{5x^2-4} dx$ | (3) $\int \frac{e^{\frac{1}{x}}}{x^2} dx$ |
| (4) $\int \frac{\ln x}{x} dx$ | (5) $\int \tan x dx$ | (6) $\int x^2 3^{5x^3+1} dx$ |
| (7) $\int \sqrt{\frac{\arcsin x}{1-x^2}} dx$ | (8) $\int \frac{x}{\sqrt{1-x^4}} dx$ | (9) $\int \frac{3x+4}{\sqrt{3x^2+8x+3}} dx$ |
| (10) $\int x^2\sqrt{3-x} dx$ | (11) $\int \sqrt{1+2x} dx$ | (12) $\int \frac{4x}{\sqrt{3x}} dx$ |
| (13) $\int x\sqrt{2x^2+7} dx$ | (14) $\int (2x^{\frac{3}{2}}+1)\sqrt{x} dx$ | (15) $\int \frac{\sin 3x}{\sqrt{5+\cos 3x}} dx$ |
| (16) $\int \frac{e^x}{4-e^x} dx$ | (17) $\int \frac{2x-3}{x^2-3x+5} dx$ | (18) $\int x\sqrt{x^2+9} dx$ |
| (19) $\int xe^{x^2} dx$ | (20) $\int \sin^2 x \cos x dx$ | (21) $\int (x+5)^6 dx$ |
| (22) $\int (3x-2)^9 dx$ | (23) $\int \frac{3x^2}{\sqrt{x^3+7}} dx$ | (24) $\int \frac{(\ln x)^5}{x} dx$ |
| (25) $\int \frac{1}{x \ln x} dx$ | (26) $\int \tan^3 x \sec^2 x dx$ | (27) $\int (2x+5)(x^2+5x)^7 dx$ |
| (28) $\int (3-4x)^8 dx$ | (29) $\int \frac{x^3}{(1+x^4)^{\frac{1}{3}}} dx$ | (30) $\int \frac{\cos(5x)}{e^{\sin(5x)}} dx$ |

2. Evaluate the following indefinite integrals.

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|---------------------------------------|---|---------------------------------------|
| (1) $\int (x+3)(x-1)^5 dx$ | (2) $\int x\sqrt{4-x} dx$ | (3) $\int \frac{x+5}{2x+3} dx$ |
| (4) $\int \frac{x^2+4}{x+2} dx$ | (5) $\int \frac{(3+\ln x)^2(2-\ln x)}{4x} dx$ | (6) $\int \sqrt{4-\sqrt{x}} dx$ |
| (7) $\int \sec^2(2x-3) dx$ | (8) $\int \frac{(2-\sqrt{x})^5}{\sqrt{x}} dx$ | (9) $\int x^2 e^{-4x^3} dx$ |
| (10) $\int e^x e^x dx$ | (11) $\int \frac{1+e^{3x}}{e^{3x}+3x} dx$ | (12) $\int \frac{\sin(\ln ax)}{x} dx$ |
| (13) $\int \frac{4x+8}{x^2+4x-25} dx$ | (14) $\int \frac{x+1}{\sqrt{x-1}} dx$ | (15) $\int x(x+1)^8 dx$ |

ANSWERS

1.

$$(1) \quad -\frac{1}{3(1+x^3)} + C$$

$$(2) \quad \frac{2}{3}(5x^2 - 4)^{\frac{3}{2}} + C$$

$$(3) \quad -e^{\frac{1}{x}} + C$$

$$(4) \quad \frac{(\ln x)^2}{2} + C$$

$$(5) \quad \ln |\sec x| + C$$

$$(6) \quad \frac{1}{15 \ln 3} 3^{5x^3+1} + C$$

$$(7) \quad \frac{2}{3}(\arcsin x)^{\frac{3}{2}} + C$$

$$(8) \quad \frac{1}{2} \arcsin x^2 + C$$

$$(9) \quad \sqrt{3x^2 + 8x + 3} + C$$

$$(10) \quad -\frac{2}{7}(3-x)^{\frac{7}{2}} + \frac{12}{5}(3-x)^{\frac{5}{2}} - 6(3-x)^{\frac{3}{2}} + C$$

$$(11) \quad \frac{1}{3}(1+2x)^{\frac{3}{2}} + C$$

$$(12) \quad \frac{8}{3\sqrt{3}}x^{\frac{3}{2}} + C$$

$$(13) \quad \frac{1}{6}(2x^2 + 7)^{\frac{3}{2}} + C$$

$$(14) \quad \frac{2}{3}(x^3 + x^{\frac{3}{2}}) + C$$

$$(15) \quad -\frac{2}{3}\sqrt{5 + \cos 3x} + C$$

$$(16) \quad -\ln |4 - e^x| + C$$

$$(17) \quad \ln |x^2 - 3x + 5| + C$$

$$(18) \quad \frac{1}{3}(x^2 + 9)^{\frac{3}{2}} + C$$

$$(19) \quad \frac{1}{2}e^{x^2} + C$$

$$(20) \quad \frac{1}{3} \sin^3 x + C$$

$$(21) \quad \frac{1}{7}(x+5)^7 + C$$

$$(22) \quad \frac{1}{30}(3x-2)^{10} + C$$

$$(23) \quad 2\sqrt{x^3 + 7} + C$$

$$(24) \quad \frac{1}{6}(\ln x)^6 + C$$

$$(25) \quad \ln |\ln x| + C$$

$$(26) \quad \frac{1}{4} \tan^4 x + C$$

$$(27) \quad \frac{1}{8}(x^2 + 5x)^8 + C$$

$$(28) \quad -\frac{1}{36}(3-4x)^9 + C$$

$$(29) \quad \frac{3}{8}(1+x^4)^{\frac{2}{3}} + C$$

$$(30) \quad -\frac{1}{5}e^{-\sin 5x} + C$$

2. (1) $\frac{1}{7}(x-1)^7 + \frac{2}{3}(x-1)^6 + C$
- (2) $\frac{2}{5}(4-x)^{\frac{5}{2}} - \frac{8}{3}(4-x)^{\frac{3}{2}} + C$
- (3) $\frac{1}{4}(2x+3+7\ln|2x+3|) + C$
- (4) $\frac{1}{2}(x^2-4x-12) + 8\ln|x+2| + C$
- (5) $\frac{9}{2}\ln|x| + \frac{3}{8}(\ln|x|)^2 - \frac{1}{3}(\ln|x|)^3 - \frac{1}{16}(\ln|x|)^4 + C$
- (6) $\frac{4}{5}(4-\sqrt{x})^{\frac{5}{2}} - \frac{16}{3}(4-\sqrt{x})^{\frac{3}{2}} + C$
- (7) $\frac{1}{2}\tan(2x-3) + C$
- (8) $-\frac{1}{3}(2-\sqrt{x})^6 + C$
- (9) $-\frac{1}{12}e^{-4x^3} + C$
- (10) $\frac{1}{2}e^{2x} + C$
- (11) $\frac{1}{3}\ln|e^{3x}+3x| + C$
- (12) $-\cos(\ln ax) + C$
- (13) $2\ln|x^2+4x-25| + C$
- (14) $\frac{2}{3}(x-1)^{\frac{3}{2}} + 4\sqrt{x-1} + C$
- (15) $\frac{1}{90}(x+1)^9(9x-1) + C$