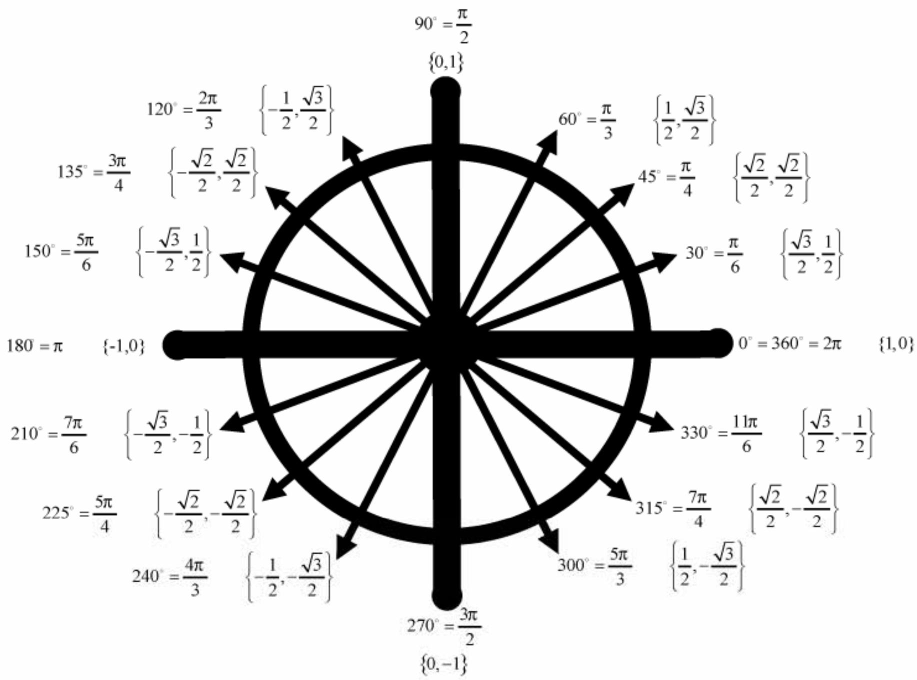
Pure Math 30:

TRIGONOMETRY I



LESSON ONE

Types of Angles

Pure Math 30: EXPLAINED!

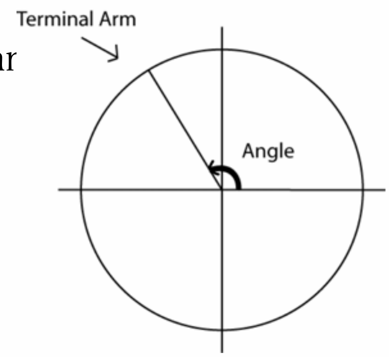
By Barry Mabillard

TRIGONOMETRY LESSON ONE

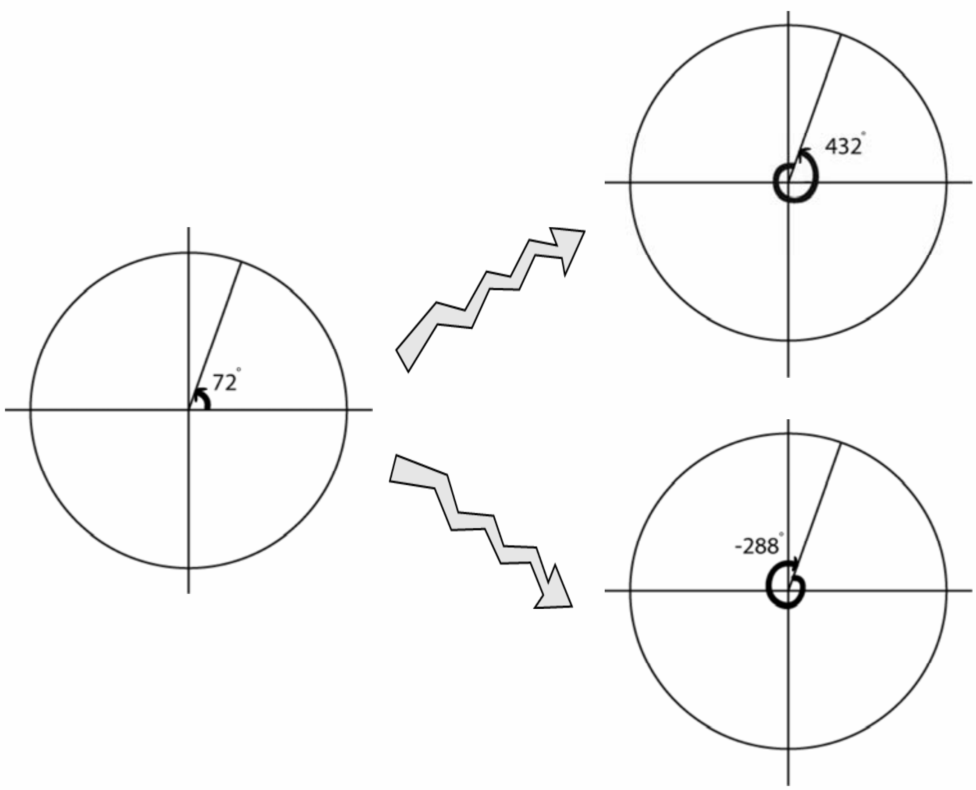
Part I Types of Angles

Memorize the following definitions:

Terminal arm: The line representing where the angle is located in a plane. The terminal arm always comes out of the origin.
Standard Angle: The angle between the positive x-axis and the terminal arm, rotating counterclockwise.
Principal angle: An angle between 0° and 360°.
Co-terminal angle: An angle less than 0° or greater thar 360° that shares the same position as a principal angle.
Negative angle: Angles obtained from rotating the terminal arm clockwise.
Reference angle: The angle between the terminal arm and the x-axis.
General Solution: A formula that gives all possible co-terminal angles.



Example 1: Find both a positive & negative co-terminal angle for 72°.

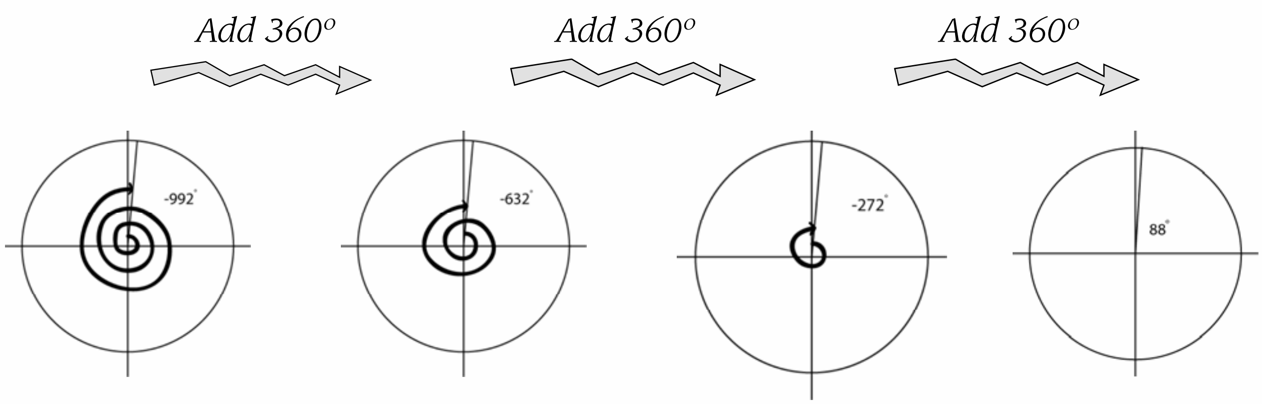


To find the positive co-terminal angle, add 360° to make one complete counterclockwise revolution.
$72^\circ + 360^\circ = 432^\circ$

To find the negative co-terminal angle, subtract 360° to make one complete clockwise revolution.
$72^\circ - 360^\circ = -288^\circ$

Example 2: Given the co-terminal angle -992°, find the principal angle.

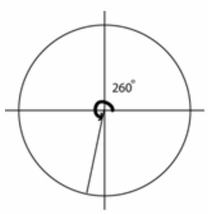
We need to "unwind" our way back to between 0° and 360° by making revolutions of 360°.



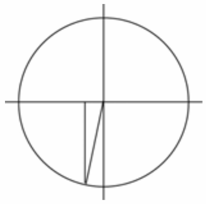
The principal angle is 88°

Example 3: Find the reference angle for 620°.

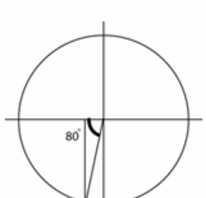
First find the principal angle. 620 ° - 360° = 260°



Connect a line to the x-axis.



Now determine the angle inside the triangle.
(260° - 180° = 80°)



The reference angle is 80°

Example 4: Find the general solution for all co-terminal angles of 101°, and state the value of the 14th positive co-terminal angle.

The general solution must include all possible revolutions of 360°, so we use the formula:

Principal Angle ± n(360°)

For this question, it's 101° ± n(360°)

When n = 1, we can either add 360° to get a positive co-terminal, or subtract 360° for a negative co-terminal.

When n = 2, we can either add 720° to get a positive co-terminal, or subtract 720° for a negative co-terminal.

The 14th positive co-terminal angle occurs when n = 14.

101° + 14(360°)
*choose the plus since we want the positive angle.

= 101° + 5040°

= 5141°

TRIGONOMETRY LESSON ONE

Part I Types of Angles

1) For each of the following principal angles, state a negative & positive co-terminal angle:

a) 47°
b) 102°
c) 321°
d) 225°

2) For each of the following co-terminal angles, find the principal angle:

a) -1023°
b) 541°
c) 888°
d) -361°

3) Find the reference angle for each of the following:

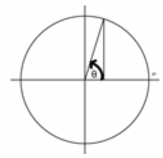
a) 992°
b) -502°
c) -1337°
d) 600°

4) Find the principal angle for each of the following reference angles:

a) 43°, Quadrant III
b) 72°, Quadrant IV
c) 89°, Quadrant II
d) 1°, Quadrant III

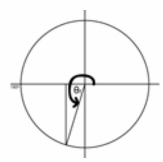
Reference Angles Quickchart:

Q1: Reference = Principal

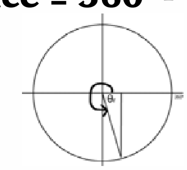


Q2: Reference = 180° – Principal

Q3: Reference = Principal – 180°



Q4: Reference = 360° - Principal



5) State the general solution for all co-terminal angles, and find the 10th positive angle for the following:

a) 52°
b) 131°
c) 300°

Answers

1.
a) -313° & 407°
b) -258° & 462°
c) -39° & 681°
d) -135° & 585°

2.
a) 57°
b) 181°
c) 168°
d) 359°

3.
a) 88°
b) 38°
c) 77°
d) 60°

4.
a) 223°
b) 288°
c) 91°
d) 181°

5.
a) 3652°
b) 3731°
c) 3900°

Part II Angle Conversions

There are three different ways of expressing angles:

Degrees: There are 360° in a circle.
Radian Decimals: 1 radian is equal to 57.3°. There are 6.28 radians in a circle.
Radian Fraction: Expressing radian values as a fraction involving π.
There are 2π radians in a circle.

Throughout the unit, you will be working with all three types of angles.
The following examples show how to convert from one type to another.

Example 1: Convert 201° to a radian decimal:

To convert from degrees → radian decimals, multiply by the fraction $\frac{\pi}{180^\circ}$

$$201^\circ \times \frac{\pi}{180^\circ} = 3.51 \text{ rad}$$

Example 2: Convert 225° to a radian fraction.

To convert from degrees → radian fractions, multiply by the fraction $\frac{\pi}{180^\circ}$

$$225^\circ \times \frac{\pi}{180^\circ} = \frac{\boxed{225^\circ}\times\pi}{\boxed{180^\circ}} = \frac{5\pi}{4}$$

Divide these numbers in your TI-83, then Math --> Frac

Remember These Conversions:

Radians → Degrees: Multiply by $\frac{180^0}{\pi}$

Degrees → Radians: Multiply by $\frac{\pi}{180^0}$

Example 3: Convert 3.24 radians to degrees:

To convert from radian decimal → degrees, multiply by the fraction $\frac{180^\circ}{\pi}$

$$3.24 \times \frac{180^\circ}{\pi} = 185.6^\circ$$

Example 4: Convert $\frac{\pi}{4}$ radians to degrees:

To convert from radian fractions → degrees, multiply by the fraction $\frac{180^\circ}{\pi}$

$$\frac{\pi}{4} \times \frac{180^\circ}{\pi} = \frac{\cancel{\pi}}{4} \times \frac{180^\circ}{\cancel{\pi}} = 45^\circ$$

<u>Example 5:</u> Convert $\frac{7\pi}{6}$ radians to a radian decimal.

To convert from radian fraction → radian decimal, simply type the fraction into your calculator:

$7 \times \pi \div 6 = 3.67$ rads

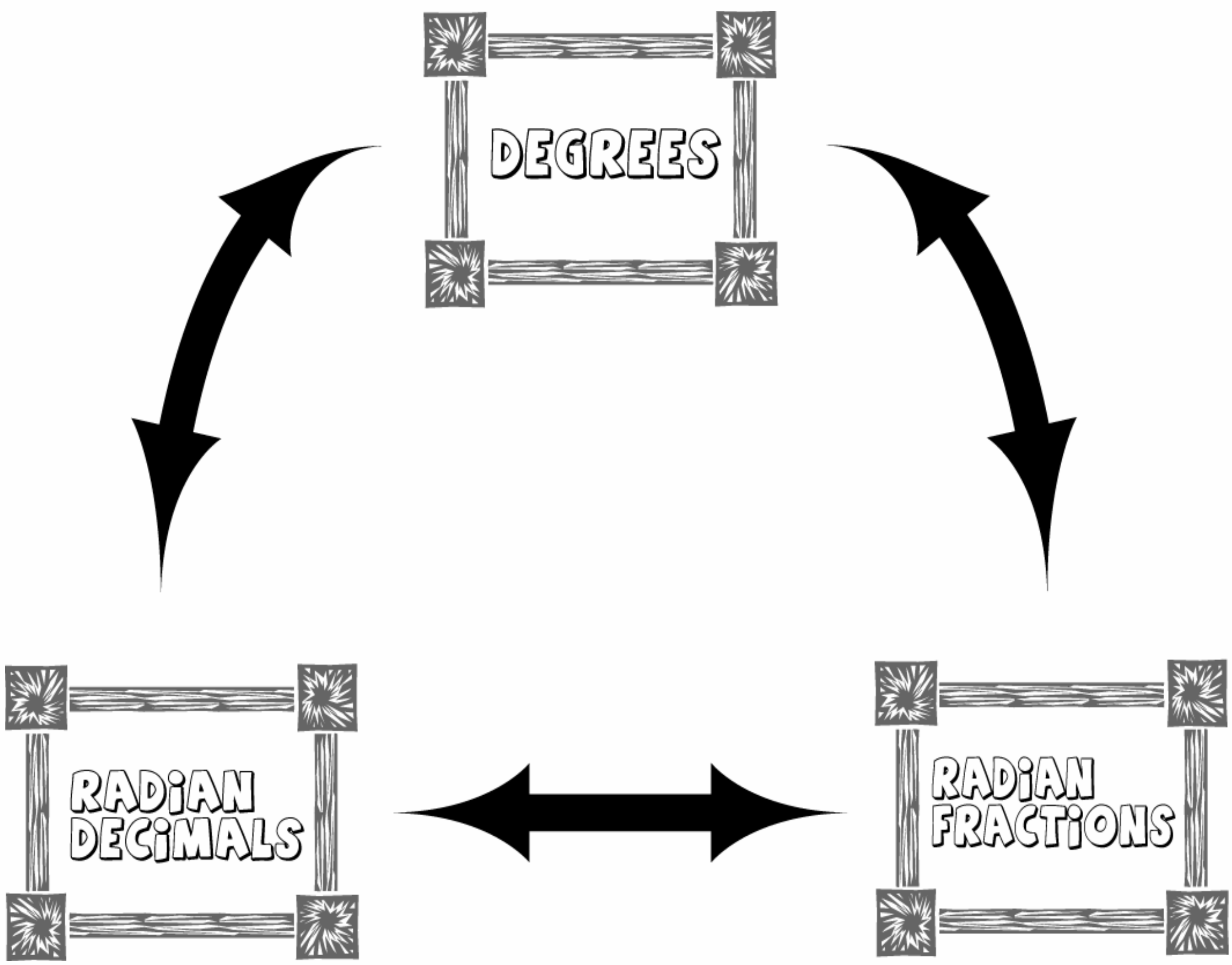
<u>Example 6:</u> Convert 0.7854 radians to a radian fraction.

To convert from radian decimal → radian fraction, first convert to degrees, then to a radian fraction.

$$0.7854 \times \frac{180^\circ}{\pi} = 45^\circ$$

$$45^\circ \times \frac{\pi}{180^\circ} = 45^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{4}$$

Divide 45° by 180° and then math → frac to get ¼.



1. Convert the following from degrees → radian decimal:

a) 342°
b) 62°
c) 100°

2. Convert the following from degrees → radian fraction:

a) 300°
b) 60°
c) 150°

3. Convert the following from radian decimal → degrees:

a) 0.02
b) 2.51
c) 1.33

4. Convert the following from radian fraction → degrees:

a) $\frac{2\pi}{3}$

b) $\frac{11\pi}{6}$

c) $\frac{3\pi}{2}$

5. Convert the following from radian fraction → radian decimal:

a) $\frac{5\pi}{4}$

b) $\frac{5\pi}{3}$

c) $\frac{\pi}{2}$

6. Convert the following from radian decimal → radian fractios:

a) 2.6180
b) 6.2832
c) 5.2360

Answers

1.
a) 5.97
b) 1.08
c) 1.75

2.
a) $\frac{5\pi}{3}$
b) $\frac{\pi}{3}$
c) $\frac{5\pi}{6}$

3.
a) 1.15°
b) 143.8°
c) 76.2°

4.
a) 120°
b) 330°
c) 270°

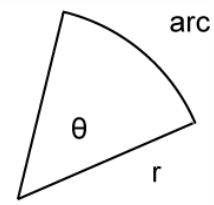
5.
a) 3.93
b) 5.24
c) 1.57

6.
a) $\frac{5\pi}{6}$
b) 2π
c) $\frac{5\pi}{3}$

TRIGONOMETRY LESSON ONE

Part III Arc Length

We can find the length of an arc by using the formula $a = r\theta$



a = length of arc
r = radius of circle
θ = angle of arc. (Radians ONLY)

Example 1: What is the length of arc if a radius of 4 m sweeps through 185.6° ?

First, convert the angle to a radian decimal. $185.6° \times \frac{\pi}{180°} = 3.24$

Next, plug your numbers into the formula
$a = r\theta$
$a = (4)(3.24)$
$a = 12.96$ m

Example 2: What is the angle in degrees if a radius of 2.3 m cuts an arc of 6.1 m ?

First solve for θ:

$\theta = \frac{a}{r}$

$\theta = \frac{6.1}{2.3}$

$\theta = 2.65$

Now convert this radian decimal to degrees: $2.65 \times \frac{180°}{\pi} = 152°$

1) Find the arc length given the following information:
a) radius = 10 m & $\theta = 5.4$ radians
b) diameter = 16 m & $\theta = 22°$

2) Find the radius given the following:
a) arc length = 16 m & $\theta = 123°$
b) arc length = 23 m & $\theta = \frac{5\pi}{4}$

3) Find the angle (in degrees) given the following:
a) arc length = 2 m & radius = 1 m
b) arc length = 5.2 m & radius = 3 m

Answers

1)
a) 54 m
b) 3.07 m

2)
a) 7.45 m
b) 5.86 m

3)
a) 114.6°
b) 99.3°